# HOMEWORK PROBLEM SET 9: DUE NOVEMBER 1, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 4.4**, **5.1**, **5.2**, and **5.3** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 9 on WeBWorK.
- **Reading for next week:** Read **Section 5.4**, **5.5**, **6.1**, and **6.2**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 4.4:** 4,8,12,16,19,25,27,32,37
- **Section 5.1:** 3cd,4d,6,9,11,13
- **Section 5.2:** 4,10,12
- **Section 5.3:** 3,4f,7,8,12,13,15

The following exercises are to be handed in for grade in lecture on the due date above:

**Exercise 1.** Let $\mathbf{F}\colon\mathbb{R}^3 \to \mathbb{R}^3$ be a $C^2$-vector field, and $f\colon\mathbb{R}^3 \to \mathbb{R}$ be a $C^2$-function. Then the product $f\mathbf{F}\colon\mathbb{R}^3 \to \mathbb{R}$ is again a $C^2$-vector field, defined as $(f\mathbf{F})(\mathbf{x}) = f(\mathbf{x})\mathbf{F}(\mathbf{x})$. Prove that

$$\mathbf{curl}(f\mathbf{F}) = f\,\mathbf{curl}(\mathbf{F}) + \nabla f \times \mathbf{F}.$$

Note that this is item number 10 in the list of Basic Identities of Vector Analysis, on page 255 of the text. (Gradient, vector fields, curl, vector analysis)

**Exercise 2.** Let $\mathbf{F}(x, y, z) = 3x^2y\,\mathbf{i} + (x^3 + y^3)\,\mathbf{j}$. Do the following:

**(a)** Show that $\mathbf{F}$ is irrotational.

**(b)** Find a function $f\colon\mathbb{R}^3 \to \mathbb{R}$, where $\mathbf{F} = \nabla f$. Note that integrating a function of more than one variable with respect to one variable is call *partial integration*. I mentioned this in class, and there is a quick writeup here:

`http://mathonline.wikidot.com/partial-integration`.

(Vector fields, curl, gradient, irrotational vector fields, conservative vector fields)

**Exercise 3.** Let $f$ be a continuous function on the interval $[a, b]$ and $g$ continuous on $[c, d]$. Show that

$$\iint_{\mathcal{R}} [f(x)g(y)]\; dA = \left[\int_a^b f(x)\,dx\right]\left[\int_c^d g(y)\,dy\right],$$

where $\mathcal{R} = [a, b] \times [c, d]$. (double integrals, the definite integral, rectangular regions, notation)