HOMEWORK PROBLEM SET 8: DUE OCTOBER 25, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 4.2** and **4.3** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 8 on WeBWorK.
- **Reading for next week:** Read **Section 4.4**, **5.1**, **5.2**, and **5.3**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 4.2:** 5,8,10,14
- **Section 4.3:** 9,10,13,16,19,21,22

The two following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. Consider a C^1-path $\mathbf{c}\colon [a,b] \to \mathbb{R}^3$. Create a new parameterization for this path in the following way: Given a C^1-function $\alpha\colon [a,b] \to [a_0,b_0]$ that is strictly increasing, one-to-one, and onto. This means that for each $s \in [a_0,b_0]$, there is a unique $t \in [a,b]$ so that $\alpha(t) = s$. Define a new C^1-path $\mathbf{d}\colon [a_0,b_0] \to \mathbb{R}^3$ by the formula $\mathbf{d}(s) = \mathbf{d}(\alpha(t)) = \mathbf{c}(t)$. So the original curve $\mathbf{c}$ is equal to the composition $(\mathbf{d} \circ \alpha)$. Do the following:

(a) Argue that the image curves of $\mathbf{c}$ and $\mathbf{d}$ are the same.

(b) Use the chain rule to express the velocity vector for $\mathbf{c}$ at time t in terms of the velocity vector for $\mathbf{d}$.

(c) Show that $\mathbf{c}$ and $\mathbf{d}$ have the same arc length.

(d) Define $\alpha(t) = \displaystyle\int_a^t ||\mathbf{c}'(\tau)||\, d\tau$ and define $\mathbf{d}$ as above by $\mathbf{d}(s) = \mathbf{c}(t)$ for the unique t so that $\alpha(t) = s$. Show that $||\mathbf{d}'(s)|| = 1$ for all s. Note that a path $\mathbf{c}\colon [a,b] \to \mathbb{R}^3$ is parameterized by arc length if $||\mathbf{c}'(t)|| = 1$ for all t. So the path $\mathbf{d}$ defined in **part(c)** is a path parameterized by arc length. We also say here that the new path $\mathbf{d}$ is a ***reparameterization*** of $\mathbf{c}$.

(curves, arc-length, parameterizations, arc-length parameter)

Exercise 2. Let $\mathbf{c}(t)$ be a flow line of a gradient field $\mathbf{F} = -\nabla V$, for $V\colon \mathbb{R}^n \to \mathbb{R}$ a C^1-potential function. Prove that V, evaluated along $\mathbf{c}$ is a decreasing function of t. (vector fields, gradient field, integral curve (flow line), potentials)