HOMEWORK PROBLEM SET 7: DUE OCTOBER 21, 2019

AS.110.202 CALCULUS III
PROFESSOR RICHARD BROWN

The following problem set is based on **Sections 3.4** and **4.1** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 7 on WeBWorK.
- **Reading for next week:** Read **Section 4.2** and **4.3**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 3.4:** 4,6,9,12,17,22
- **Section 4.1:** 3,8,9,12,23,25

The three following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. Determine the extrema of $f(x,y) = \sqrt{x} + 8\sqrt{y}$ subject to the constraint $x^2 + y^2 = 17$, where $x \geq 0$ and $y \geq 0$.

(a) Explain why f must attain both a global maximum and a global minimum along the given constraint curve.

(b) Solve the resulting Lagrange multiplier system $\nabla f(x,y) = \lambda \nabla g(x,y)$, $g(x,y) = 0$. Explain why you only see one critical point of f along this constraint curve.

(c) Identify the global minimum and global maximum of f subject to the constraint.

(extrema, Lagrange multipliers, constraints, domains)

Exercise 2. Determine the extrema of $f(x,y) = x$ subject to the constraint $y^2 - 4x^3 + 4x^4 = 0$.

(a) Explain why f must attain both a global maximum and a global minimum along the given constraint curve.

(b) Solve the resulting Lagrange multiplier system $\nabla f(x,y) = \lambda \nabla g(x,y)$, $g(x,y) = 0$. Here, you should see multiple critical points of f along the constraint curve.

(c) Use technology to graph the constraint curve, and use the graph to identify the global extrema of f.

(d) Compare your result in **part (b)** with what you found in **part (c)**. What accounts for any differences that you observed?

(extrema, Lagrange multipliers, constraints, domains)

Exercise 3. A particle moves in $\mathbb{R}^3$ so that its acceleration is a constant $-\mathbf{k}$. The particle's initial position at $t = 0$ is $(-1, 0, 2)$ and its velocity at $t = 0$ is the vector $\mathbf{i} + \mathbf{j}$. Do the following:

(a) When does the particle hit the $z = 0$ plane (the floor?)

(b) Where does it hit the floor?

(c) How high does the particle travel during its flight?

(d) How long is the particle's path from $t = 0$ to when it hits the floor?

(curves, local extrema, arclength, curve velocity and acceleration)