HOMEWORK PROBLEM SET 6: DUE OCTOBER 11, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 3.2** and **3.3** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 6 on WeBWorK.
- **Reading for next week:** Read **Section 3.4** and **4.1**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 3.2:** 1,2,3,4,9,10
- **Section 3.3:** 4,12,13,20,21,28,37

The two following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. Suppose you are asked to find minimum and maximum values of a continuous function $f\colon \mathbb{R}^n \to \mathbb{R}$ on one of the following regions:

(a) all of $\mathbb{R}^n$,

(b) an open region in $\mathbb{R}^n$ (one example is region $x_1^2 + \ldots + x_n^2 < 1$),

(c) a hypersurface in $\mathbb{R}^n$ comprised of those points satisfying a single constraint equation $g(x_1, \ldots, x_n) = c$ (examples include the hypersurface defined by $x_1^2 + \ldots + x_n^2 = 1$ or $x_1 + \ldots + x_n = 0$), or

(d) a closed and bounded region in $\mathbb{R}^n$ (an example is the region defined by $x_1^2 + \ldots + x_n^2 \leq 1$).

For each of the four types of regions described above: (1) state and defend whether or not a global minimum or global maximum value necessarily exists (either by quoting a theorem from the text that global extrema exist or by giving a specific example of a function that has no global extremum on a particular region type), and (2) explain, in your own words, the procedure for finding the global extrema in the case(s) where they do exist. (definitions, extrema, regions, procedures)

Exercise 2. Do the following:

(a) Let f be a C_1 function on the real line $\mathbb{R}$. Suppose that f has exactly one critical point x_0, that is a local minimum for f. Show that x_0 is also then a *global minimum* for f; That is, show $f(x) \geq f(x_0)$, for all $x \in \mathbb{R}$.

(b) To see, through the following example, that this is not necessarily true for functions defined on more than one variable, define $f\colon \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = -y^4 - e^{-x^2} + 2y^2\sqrt{e^x + e^{-x^2}}.$$

Show that (i) the origin is the only critical point, and that it is a local minimum. Then argue that (ii) f has no global minimum.

(global extrema, local extrema, functions on $\mathbb{R}$, functions on $\mathbb{R}^n$)