HOMEWORK PROBLEM SET 5: DUE OCTOBER 4, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 2.6** and **3.1** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 5 on WeBWorK.
- **Reading for next week:** Read **Section 3.2** and **3.3**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 2.6:** 2ac,3a,4,6,8a,9a,10a,13,17,18
- **Section 3.1:** 1,3,6,9,11,16

The three following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. For $f(x, y) = y^2 - x^2$, do the following:

(a) Draw some level sets for f (basically, draw a contour map, as in the example on page 78 of the text).

(b) Draw some representative vertical sections (slices).

(c) Use **parts (a)**, and **(b)** to draw a graph of f.

(d) Compute $\nabla f(2, 0)$ and $\nabla f(0, 2)$. What information does each of these quantities convey about the graph of f?

(e) In light of your answer to **part (c)**, speculate as to the meaning of the equation $\nabla f(0, 0) = \mathbf{0}$.

(definitions, gradients, graphs of functions, level sets and slices)

Exercise 2. For $g(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$, do the following:

(a) In what direction is the directional derivative of g at $(1, 1)$ equal to 0?

(b) In what direction is the directional derivative of g at (x_0, y_0) equal to 0, for (x_0, y_0) a point in the first quadrant of the plane?

(c) Describe the level sets of g. In particular, discuss them in terms of the result of **part (b)**.

(directional derivative, gradient, level sets)

Exercise 3. Let

$$h(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

The graph of h is in the figure. Do the following:

(a) For $(x,y) \neq (0,0)$, calculate $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$.

(b) Show that $\frac{\partial h}{\partial x}(0,0) = 0 = \frac{\partial h}{\partial y}(0,0)$.

(c) Show that $\frac{\partial^2 h}{\partial x \partial y}(0,0) = 1$, and $\frac{\partial^2 h}{\partial y \partial x}(0,0) = -1$.

(d) Explain what went wrong and why the second mixed partials here are not equal. (iterated derivatives, C^2-functions)

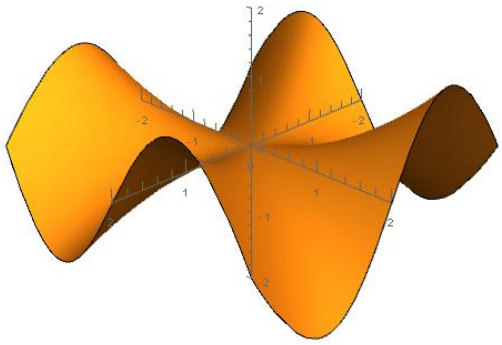

FIGURE 1. The graph of $h(x,y)$ in **Exercise 3**.