HOMEWORK PROBLEM SET 2: DUE SEPTEMBER 13, 2019

AS.110.202 CALCULUS III
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Question 1. Do WeBWorK Problem Set 2.

Question 2. Do the following:

(a) Find a normal vector $\mathbf{n}$ to the plane $x - 2y + 3z - 1 = 0$ in $\mathbb{R}^3$ (Specifying a vector includes both specifying its components as well as where it is based), and explain how you know that this vector is perpendicular to any vector $\mathbf{v}$ that lies in the plane.

(b) Let A, B, C, and D be real numbers with at least one of A, B, or C non-zero. Find a normal vector $\mathbf{n}$ to the plane $Ax + By + Cz + D = 0$ in R^3 and explain how you know that this vector is orthogonal to any vector $\mathbf{v}$ that lies in the plane.

(c) Considering the general case in **part (b)**, explain why it is the case that the vector $\mathbf{n} \times \mathbf{v}$ always lies in the plane defined by the equation.

(definitions, viewpoint)

Question 3. Consider the set of all solutions to the system of equations

$$\begin{aligned} x + 2y + 3z + u &= 4 \\ 5x - 3y + 2z - 6u &= 0 \end{aligned}$$

as a subset of $\mathbb{R}^4$. Do the following:

(a) Explain how you can or could determine that this solution set defines a plane in $\mathbb{R}^4$. Include in this explanation what the solution set may look like if it is not a plane in $\mathbb{R}^4$.

(b) Parameterize the plane defined by the system of equations above, using the parameter variables s and t.

(c) Describe the space that this system determines if these equations were describing a subset of $\mathbb{R}^5$, with coordinates (x, y, z, u, v). How can we adapt the parameterization from **part (b)** to parameterize this new space in $\mathbb{R}^5$.

(subsets of real space)

Question 4. Consider a mapping that takes every nonzero vector $\mathbf{x} \in \mathbb{R}^3$ to the vector of length 4 that points in the opposite direction of $\mathbf{x}$. Write an expression for this mapping, both symbolically (determining the domain and range), and via an expression (including the component functions). (descriptions of functions)

Question 5. Find the matrix A so that the function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2$, defined by $\mathbf{f}(\mathbf{x}) = A\mathbf{x}$, satisfies

$$\mathbf{f}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{f}\left(\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{f}\left(\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

Show that $\mathbf{f}$ is a *linear* function. (matrices, vectors, linear functions)