HOMEWORK PROBLEM SET 13: NOT TO BE HANDED IN....

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 8.1**, **8.2**, **8.3**, and **8.4** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 13 on WeBWorK.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 8.1:** 8,11a,15,22
- **Section 8.2:** 1,5,8,15,18
- **Section 8.3:** 2,3,8,9,11,17,19
- **Section 8.4:** 1,3,9a,11,12,14

The following exercise are offered only for exploration of the concepts in Chapter 8:

Exercise 1. Do the following:

(a) Parameterize the unit sphere $\mathcal{S}_1 \subset \mathbb{R}^3$ using the spherical coordinates (ρ, θ, φ) by setting $\rho = 1$. Then determine the orientation of your parameterized sphere by calculating the unit normal at the north pole $(x, y, z) = (0, 0, 1)$.

(b) Reparameterize $\mathcal{S}_1$ so that the unit normal at the north pole points in the other direction. Hint: This can be achieved in many ways. For example, by switching the expressions for x and y, or by reversing the direction along one axis by negating the expression.

(c) Calculate the scalar line integral of the vector field $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ along the equator by orienting it according to each of your two parameterizations above (really, just set the z-expression to 0, thereby fixing φ, and then writing the equator curve as a function of θ alone.) Explain geometrically why this quantity is 0 for both (indeed, all) parameterizations.

(d) Calculate the flux of $\mathbf{F}$ through $\mathcal{S}_1$, as a vector surface integral, under both parameterizations.

(e) Integrate the divergence of $\mathbf{F}$ throughout the closed, unit ball $\mathcal{B}_1$, consisting of $\mathcal{S}_1$ and its interior, for each parameterization (now ρ is not a constant 1 anymore). Explain why Gauss' Theorem is not violated in both cases, even though your two parameterizations in **part (d)** will yield different answers.

((Re)parameterized surfaces, definitions, surface normal, Gauss' Theorem)