HOMEWORK PROBLEM SET 12: DUE NOVEMBER 22, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 7.3**, **7.4**, **7.5**, and **7.6** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 12 on WeBWorK.
- **Reading for next week:** Read **Section 8.1**, and **8.2**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 7.3:** 2,5,8,14,18,23
- **Section 7.4:** 3,5,25
- **Section 7.5:** 3,5,12,20
- **Section 7.6:** 2,4,6,13

The following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. Do the following:

(a) Given the parameterization $\Phi\colon \mathcal{D} \subset \mathbb{R}^2 \to \mathbb{R}^3$,
$$\Phi(u,v) = \left(x(u,v), y(u,v), z(u,v)\right),$$
show that
$$||\mathbf{T}_u \times \mathbf{T}_v|| = \sqrt{\left[\frac{\partial(x,y)}{\partial(u,v)}\right]^2 + \left[\frac{\partial(x,z)}{\partial(u,v)}\right]^2 + \left[\frac{\partial(y,z)}{\partial(u,v)}\right]^2}.$$

(b) Given the parameterization of $\mathcal{S}_{r_0} \subset \mathbb{R}^3$, the sphere of radius $r_0 > 0$,
$$x = r_0 \cos\theta \sin\varphi, \quad y = r_0 \sin\theta \sin\varphi, \quad z = r_0 \cos\varphi,$$
where $\theta \in [0, 2\pi]$, and $\varphi \in [0, \pi]$, verify the formula
$$||\mathbf{T}_\theta \times \mathbf{T}_\varphi||\; d\theta\, d\varphi = r_0^2 \sin\varphi\, d\theta\, d\varphi.$$

(Parameterized surfaces, definitions, surface normal, infinitesimal area)

Exercise 2. Let $\mathcal{C} \subset \mathbb{R}^3$ be the cylinder of radius r_0, centered on the z-axis between the planes $z = 0$ and $z = 10$. Do the following:

(a) Parameterize $\mathcal{C}$ as a surface using the cylindrical coordinates θ and z on $\mathbb{R}^3$.

(b) Compute $\mathbf{T}_\theta$ and $\mathbf{T}_z$, along with $\mathbf{T}_\theta \times \mathbf{T}_z$ and $||\mathbf{T}_\theta \times \mathbf{T}_z||$.

(c) Draw $\mathcal{C}$ along with $\mathbf{T}_\theta$ and $\mathbf{T}_z$ at a suitable point $(\theta_0, z_0) \in \mathcal{C}$.

(d) Compute the surface area of $\mathcal{C}$ using a scalar surface integral.

(e) Calculate the flux of the vector field $\mathbf{F}(\mathbf{x}) = a\,\mathbf{i} + b\,\mathbf{j}$, for $a, b \in \mathbb{R}$. Explain why the flux of $\mathbf{F}$ through $\mathcal{C}$ is always 0 for all real values of a and b.

(scalar line integral, vector line integral, curves, vector fields)