# HOMEWORK PROBLEM SET 11: DUE NOVEMBER 15, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 7.1**, and **7.2** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 11 on WeBWorK.
- **Reading for next week:** Read **Section 7.3**, **Section 7.4**, **Section 7.5**, and **7.6**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 7.1:** 4,6,9,11a,20,24
- **Section 7.2:** 1,4d,5,8,11,14,20

The following exercises are to be handed in for grade in lecture on the due date above:

**Exercise 1.** Suppose $\mathbf{c}_1 \colon [a_1, b_1] \to \mathbb{R}^n$ and $\mathbf{c}_2 \colon [a_2, b_2] \to \mathbb{R}^n$ are two simple $\mathbb{R}^n$-curves, where $\mathbf{c}_1(a_1) = \mathbf{p} = \mathbf{c}_2(a_2)$, and $\mathbf{c}_1(b_1) = \mathbf{q} = \mathbf{c}_2(b_2)$, but except for the start and finish, the two curves have no points in common. Let $\mathbf{F} \colon \mathbb{R}^n \to \mathbb{R}^n$ be a $C^0$-vector field. Do the following:

**(a)** State the definition of a *simple closed curve*, and show that the curve $\mathbf{c} = \mathbf{c}_1 \cup \mathbf{c}_2$ is a simple closed curve.

**(b)** Write down a piecewise-defined parameterization for the curve $\mathbf{c}$ in the following way: First travel along $\mathbf{c}_1$ and then travel along $\mathbf{c}_2$ in the opposite direction.

**(c)** Show that the two integrals

$$\int_{\mathbf{c}_1} \mathbf{F} \bullet d\mathbf{s}, \quad \text{and} \quad \int_{\mathbf{c}_2} \mathbf{F} \bullet d\mathbf{s}$$

are equal if and only if $\displaystyle\int_{\mathbf{c}} \mathbf{F} \bullet d\mathbf{s} = 0$.

(Vector line integrals, vector fields, curves, simple closed curves, definitions)

**Exercise 2.** Let $\mathbf{G}$ be a $C^0$-vector field on $\mathbb{R}^n$ and $\mathbf{d} \colon [a, b] \to \mathbb{R}^n$ a curve. Show the following:

**(a)** If $\mathbf{G}$ is perpendicular to the curve at every point, then the vector line integral of $\mathbf{G}$ along $\mathbf{d}$ is 0. That is,

$$\int_{\mathbf{d}} \mathbf{G} \bullet d\mathbf{s} = 0.$$

Note: We say here that $\mathbf{G}$ does no work on a particle moving along the curve.

**(b)** If $\mathbf{G}$ is parallel to the curve at every point, then the vector line integral of $\mathbf{G}$ along $\mathbf{d}$ equals the scalar line integral of the norm of the vector field along $\mathbf{d}$.

That is,

$$\int_{\mathbf{d}} \mathbf{G} \bullet d\mathbf{s} = \int_{\mathbf{d}} ||\mathbf{G}|| \, ds.$$

Note: A vector field $\mathbf{G}$ is parallel to a curve $\mathbf{d}$ if $\mathbf{G}\left(\mathbf{d}(t)\right) = \lambda(t)\mathbf{d}'(t)$ all along the curve, where $\lambda(t)$ is some real-valued, positive function $\lambda\colon [a, b] \to \mathbb{R}$.

(scalar line integral, vector line integral, curves, vector fields)