HOMEWORK PROBLEM SET 10: DUE NOVEMBER 8, 2019

AS.110.202 CALCULUS III
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The following problem set is based on **Sections 5.4**, **5.5**, **6.1**, and **6.2** of the text. Along with the exercises below, please do the following:

- **WeBWorK:** Complete Problem Set 10 on WeBWorK.
- **Reading for next week:** Read **Section 7.1**, and **7.2**.

For practice (neither to be handed in nor graded), here is a set of selected textbook problems:

- **Section 5.4:** 1,2,5
- **Section 5.5:** 1,5,16,18
- **Section 6.1:** 2,3,7,10,11
- **Section 6.2:** 3,5a,8,10,15,23,35

The following exercises are to be handed in for grade in lecture on the due date above:

Exercise 1. Do the following:

(a) Let $f\colon [a,b] \to \mathbb{R}$ be a non-negative function of one variable, so that $f(x) \geq 0$, for all $x \in [a,b]$. Let $\mathcal{R} \subset \mathbb{R}^2$ be the region between the x-axis and the graph of f, between the lines $x = a$ and $x = b$. Show

$$\int_a^b f(x)\,dx = \iint_{\mathcal{R}} dA.$$

(b) $f : \mathcal{D} \to \mathbb{R}$ be a non-negative function of two variables, defined on a closed, bounded region $\mathcal{D} \subset \mathbb{R}^2$, so that $f(x,y) \geq 0$, for all $(x,y) \in \mathcal{D}$. Let $\mathcal{S} \subset \mathbb{R}^3$ be the solid region between the $\mathcal{D}$ in the xy-plane and the graph of f. Show

$$\iint_{\mathcal{D}} f(x,y)\,dA = \iiint_{\mathcal{S}} dV.$$

(Multiple integration, geometry of integration)

Exercise 2. Let $\mathcal{S} = [0,1] \times [0,1]$ be the unit square in $\mathbb{R}^2$, and $\mathcal{C} = [0,1] \times [0,1] \times [0,1]$ be the unit cube in $\mathbb{R}^3$. For each map below, do the following: **(i)** Draw the image of the map, either in the xy-plane or in xyz-space; **(ii)** Calculate the Jacobian determinant of the map; **(iii)** Integrate to calculate the volume of the image of each map.

(a) $T_p\colon \mathcal{S} \to \mathbb{R}^2$, where $T_p(r,\theta) = (r\cos 2\pi\theta, r\sin 2\pi\theta)$.

(b) $T_c\colon \mathcal{C} \to \mathbb{R}^3$, where $T_c(r,\theta,z) = (r\cos 2\pi\theta, r\sin 2\pi\theta, z)$.

(c) $T_s\colon \mathcal{C} \to \mathbb{R}^3$, where $T_s(r,\theta,\varphi) = (r\cos 2\pi\theta \sin \pi\varphi, r\sin 2\pi\theta \sin \pi\varphi, r\cos \pi\varphi)$.

(d) $T_\ell\colon \mathcal{S} \to \mathbb{R}^2$, where $T_\ell(u,v) = (2u - v, u - 3v)$.

(e) $T_r\colon \mathcal{S} \to \mathbb{R}^2$, where $T_r(u,v) = (u^2 - v^2, 2uv)$.

(Change of variables, multiple integration, geometry of planar maps)