P123 2.4.6

Given a parameterization for the curves:

a) The line through (1, 2, 3) and (−2, 0, 7)

The direction vector is: $(-2,0,7)-(1,2,3)$

$=(-3,-2,4)$, Therefore the parameterization of the line is given by: $(x,y,z)-(1,2,3)=t(-3,-2,4)$

$(x-1, y-2, z-3)=(-3t, -2t, 4t)$

$\Rightarrow \quad x=-3t+1,\ y=-2t+2,\ z=4t+3$

b) The graph of $f(x)=x^2$

The parameterization is just (t, t^2)

c) The square with vertices (0, 0), (0, 1), (1, 1), (1, 0)

Use [0, 4] to parameterize the square. From (0, 0) to (0, 1), it is $t \mapsto (0, t) \quad 0 \le t < 1$;

From (0, 1) to (1, 1), it is $t \mapsto (t-1, 1) \quad 1 \le t < 2$

From (1, 1) to (1, 0), it is ~~$t \mapsto (1-(t-2))$~~

$t \mapsto (1, 1-(t-2)) = (1, 3-t) \quad 2 \le t < 3$

From (1, 0) to (0, 0) it is $t \mapsto (1-(t-3), 0)$

i.e. $t \mapsto (4-t, 0) \quad 3 \le t \le 4$

Therefore the parameterization is:

$$c(t) = \begin{cases} (0, t) & 0 \le t < 1 \\ (t-1, 1) & 1 \le t < 2 \\ (1, 3-t) & 2 \le t < 3 \\ (4-t, 0) & 3 \le t \le 4 \end{cases}$$

P116 2.3.21

Find the equition of the tagent plane to $z = x^2 + 2y^3$ at $(1,1,3)$.

Solution:

The normal vector of the surface defined by $f(x,y,z) = z - x^2 - 2y^3 = 0$ at $(1,1,3)$ is given by:

$$\nabla f \Big|_{(1,1,3)} = \left(\frac{\partial f}{\partial x}\Big|_{(1,1,3)}, \frac{\partial f}{\partial y}\Big|_{(1,1,3)}, \frac{\partial f}{\partial z}\Big|_{(1,1,3)} \right)$$

$$= \left(-2x \Big|_{(1,1,3)}, -6y^2 \Big|_{(1,1,3)}, 1 \Big|_{(1,1,3)} \right)$$

$$= (-2, -6, 1)$$

So a point $(x,y,z) \in \mathbb{R}^3$ is in the tangent plane iff the vector $(x,y,z) - (1,1,3)$ is perpendicular to $(-2,-6,1)$. Therefore:

$$(x-1, y-1, z-3) \cdot (-2,-6,1) = 0$$

The function is: $-2(x-1) - 6(y-1) + (z-3) = 0$

$$2x + 6y - z - 5 = 0$$

d) The ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Re-write the equation as $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$, Then

from the formula: $\cos^2\theta + \sin^2\theta = 1$, we find such

a parameterization: $\frac{x}{3} = \cos\theta$ $\frac{y}{5} = \sin\theta$ $(0 \le \theta < 2\pi)$

$\Rightarrow$ $c(\theta) = (3\cos\theta, 5\sin\theta)$ $(0 \le \theta < 2\pi)$