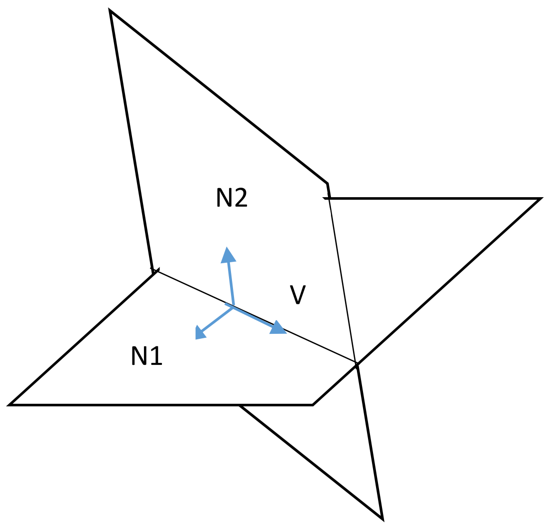
1.3.20) given two planes

$$x + 2y + z = 0 \; and \; x - 3y - z = 0$$

There are three possibilities for their intersection, empty [they are parallel], a line or the whole planes themselves. First step is to find orthogonal [Or normal if their magnitude is 1] vectors to each plane.

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \; is \; orthogonal \; to \; x + 2y + z = 0 \; and \; \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} \; is \; othogonal \; to \; x - 3y - z = 0$$

This is the immediate consequence of the equations of those planes. Because these two vectors are not parallel the planes are not parallel or equal, so their intersection is a line. As you can see in the picture for two planes with orthogonal vectors N1 and N2 the vector V which is parallel to their intersection line is also parallel to their cross product, because it should be orthogonal two both of them.



So we should continue with computing cross product of the vectors that we found.

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} = \det \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 1 & -3 & -1 \end{matrix} = \vec{i} + 2\vec{j} - 5\vec{k}$$

Now we know the vector parallel to the intersection line, for writing the equation for that line only remaining thing is to find one point [And it's not important which point] on the intersection line. So we solve the equations simultaneously.

$$x + 2y + z = x - 3y - z \;\rightarrow\; 2z = -5y, set \; z = 0 \;\rightarrow\; y = 0 \; subsituting \; in \; the \; main \; equation$$

$$gives\ us\ x = 0, so\ the\ point \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} is\ on\ the\ intersection\ line.$$

Therefor we need to write the equation for a line parallel to $\begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$ that goes through origin.

$$it\ will\ be \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} or\ equvalitnyl\ x = t\ \&\ y = 2t\ \&\ z = -5t$$