CHALLENGE PROBLEM SET: CHAPTER 2, COURSE WEEK 3

110.202 CALCULUS III
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Question 1. Describe some appropriate level surfaces and sections of the graphs of

- **(a)** $f(x,y,z) = 2x^2 + y^2 + z^2$
- **(b)** $f(x,y,z) = x^2$
- **(c)** $f(x,y,z) = xyz$.

Question 2. Compute the following limits if they exist:

- **(a)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{xy}-1}{y}$
- **(b)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{x^2y^2}$
- **(c)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2+2}$

Question 3. Compute the following limits if they exist:

- **(a)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy}$
- **(b)** $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(xyz)-1}{xyz}$
- **(c)** $\displaystyle\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, where $f(x,y,z) = \dfrac{x^2+3y^2}{x+1}$

Question 4. Do the following:

- **(a)** Can $\frac{\sin(x+y)}{x+y}$ be made continuous by suitably defining it at $(0,0)$?
- **(b)** Can $\frac{xy}{x^2+y^2}$ be made continuous by suitably defining it at $(0,0)$?
- **(c)** Prove that $f : \mathbb{R}^2 \to \mathbb{R}$, $(x,y) \mapsto ye^x + \sin x + (xy)^4$ is continuous.

Question 5. Let

$$f(x,y) = \begin{cases} \dfrac{x^2y^4}{x^4+6x^8} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

- **(a)** Show that $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ exist.
- **(b)** Show that f is not continuous at $(0,0)$ by showing that f is not continuous at $(0,0)$.

Question 6. Compute the tangent plane at $(1,0)$ for each of the following functions:

- **(a)** $f(x,y) = xe^{(-x^2-y^2)}$
- **(b)** $f(x,y) = \dfrac{xy}{(x^2+y^2)}$
- **(c)** $f(y,z) = z^2\cos y$.