

For an answer, we "play" with the directional derivative:

$$\left. \frac{d}{dt} f(\vec{x}_0 + t\vec{v}) \right|_{t=0} = \dots = \nabla f(\vec{x}_0) \cdot \vec{v}$$

for $\vec{v} \in \mathbb{R}^n$ chosen so that $\|\vec{v}\| = 1$.

For any choice of $\vec{v}$, the size of the dir. der. is

$$\nabla f(\vec{x}_0) \cdot \vec{v} = \|\nabla f(\vec{x}_0)\| \cdot \|\vec{v}\| \cos \theta$$

(this is the dot product, where θ is the angle between $\nabla f(\vec{x}_0)$ and $\vec{v}$ as vectors in $\mathbb{R}^n$).

And since $\|\vec{v}\| = 1$,

$$\nabla f(\vec{x}_0) \cdot \vec{v} = \|\nabla f(\vec{x}_0)\| \cos \theta$$

For different choices of $\vec{v}$, the only thing that changes is the angle θ .

Here $\cos \theta$ is the largest when $\theta = 0$: when $\nabla f(\vec{x}_0)$ and $\vec{v}$ are parallel!

Caution: This only works when $\nabla f(\vec{x}_0) \neq \vec{0}$.

Keep this in mind.

P.137

Thm Given $f: U \subset \mathbb{R}^n \rightarrow \mathbb{R}$ differentiable at

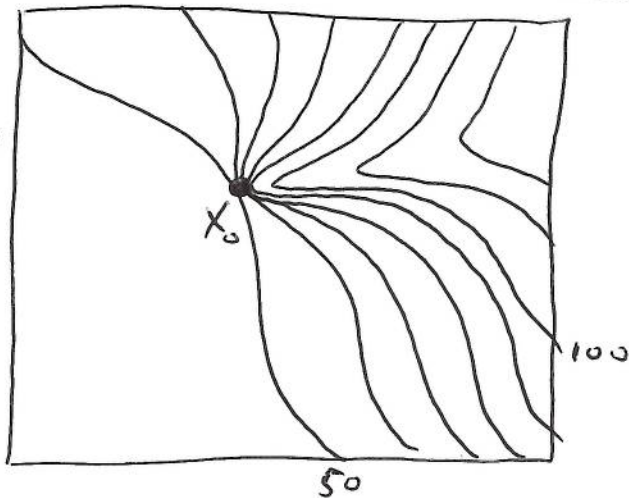
$\vec{x}_0 \in \mathbb{R}^n$ where $\nabla f(\vec{x}_0) \neq \vec{0}$. Then $\nabla f(\vec{x}_0)$

P.137

points in the direction of fastest increase of f .

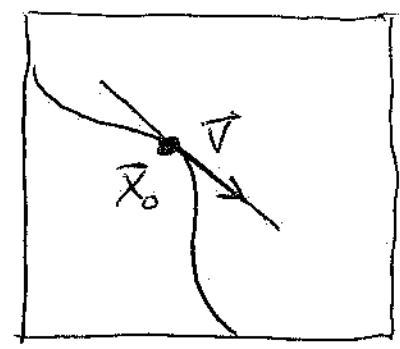
Note: This is the first very useful property of ∇f .

Think about this in the context of a topographical map (right). At $\vec{x}_0$, which direction would $\nabla f(\vec{x}_0)$ be pointing?



Q: These topographic curves are the level sets of a height function. What is the relationship between $\nabla f(\vec{x}_0)$ and the level set containing $\vec{x}_0$?

A: To see this, let $\vec{v}$ be a vector based at $\vec{x}_0$ and tangent to the level set containing $\vec{x}_0$.



(choose it to be a unit vector). Then what is the directional derivative of f at $\vec{x}_0$ in the direction of $\vec{v}$? Here f is not changing in this direction, so $\nabla f(\vec{x}_0) \cdot \vec{v} = 0$. p. 139

This is true for any $\vec{v}$ based at $\vec{x}_0$ and tangent to the level set, even in higher dimensions

Conclusion? This is extremely useful, as the gradient allows us to ~~detect~~ detect tangent directions to level sets.

