Welcome to AS.110.202 Calculus III.

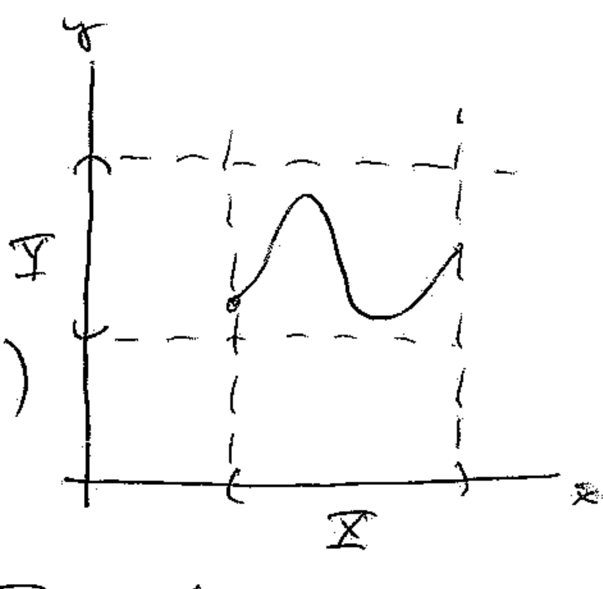
Calculus III (vector calculus, multivariable calculus) is the next step of the 2-semester (year-long) sequence of Calculus I (roughly Calculus AB) and Calculus II (roughly Calculus BC), called single variable calculus: (SVC)

SVC is the study of the properties and characteristics of the functional relationships between 2 quantifiable entities, one you have control over or know something about, and one you seek to ~~~~ know more of.

Functions relating these quantitative entities are called <u>mathematical models</u>; they help us to study, analyze, and predict behavior.

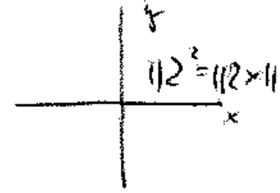
In SVC, measurable quantities are $\underbrace{\text{entities}}_{}$ (variables?) that take values in the real numbers, so as a subset of $\mathbb{R}$:

If $x \in X \subset \mathbb{R}$ (control) and $y \in Y \subset \mathbb{R}$ (the quantity we want to study)



Then the function can be represented as $f: X \to Y$, $f(x) = y$.

Then $\text{graph}(f) = \{(x,y) \in \mathbb{R}^2 \mid y = f(x)\}$ is a visual (read: geometric) representation of f in the plane $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ as the set of solutions to the equation $y = f(x)$.



Properties of interest for functions $f: X \to \mathbb{R}$ include the domain, codomain, and range ("image of f"), continuity, differentiability, critical pts, extrema, concavity, integrability, etc.

These help us to better understand the relationship between x and y.

$\text{Range}(f) = \text{Im}(f) = \{y \in \mathbb{R} \mid f(x) = y \text{ for some } x \in X\}$.

So what is multivariable calculus (MVC)?

Sometimes an entity is best analyzed in its relationship to more than 1 controllable entity.

ex: Place a flat, thin, round pan on a stove and light the burner underneath. The temp. T° of the pan will increase, but each pt of the pan at a different rate.

① Fix a moment of time t_0 and measure T° at each pt.

$\Rightarrow \quad T^\circ_t : D \to \mathbb{R}$, where $D \subset \mathbb{R}^2$ looks like a closed disk.

② If the pan is very thick, then

$T^\circ : P \to \mathbb{R}, \quad P \subset \mathbb{R}^3$

$(x,y,z) \in P \subset \mathbb{R}^3$

③ Record both density ~~time~~ and temperature as a function of position in the pan (at a fixed time) alone and you get

$$f : P \to \mathbb{R}^2, \quad P \subset \mathbb{R}^3$$

$$f(x,y,z) = \left(t(x,y,z),\ T^\circ(x,y,z)\right) \in \mathbb{R}^2.$$

Also allow time to vary (input or output?).

So how do we analyze functions like $f: \mathbb{R}^3 \to \mathbb{R}^2$? What does it mean for f to have limits, be continuous, differentiable, integrable, have extrema, etc? Where does its graph live??

That is the focus of this course! All of the things you focused on in SVC will have counterparts in MVC, but the added complexity of extra dimensions will be problematic.

For example, how do we visually represent $T^\circ: D \subset \mathbb{R}^2 \to \mathbb{R}$? And $f: P \subset \mathbb{R}^3 \to \mathbb{R}^2$?

To start, we need to well-understand places (spaces) like $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^5$, ..., so that subsets of these can serve as domains, codomains, images, etc.