Lecture Questions III: 110.202 Calculus III

Professor Richard Brown

Mathematics Department

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Question 1

For $f : \mathbb{R}^2 \to \mathbb{R}$ a C^0-function, $\displaystyle\int_0^4 \int_0^x f(x,y)\, dydx =$

- A. $\displaystyle\int_0^x \int_0^4 f(x,y)\, dxdy.$
- B. $\displaystyle\int_0^4 \int_0^y f(x,y)\, dxdy.$
- C. $\displaystyle\int_0^y \int_0^4 f(x,y)\, dydx.$
- D. $\displaystyle\int_0^4 \int_y^4 f(x,y)\, dxdy.$
- E. None of the above.

Question 2

Let $\mathbf{F}$ be a non-vanishing C^1-vector field in $\mathbb{R}^3$. Here are two statements:

1. If $\mathbf{F}$ is everywhere ________________ to a smooth, oriented curve $\mathcal{C}$, then $\displaystyle\int_{\mathcal{C}} \mathbf{F} \bullet d\mathbf{s} = 0$.
2. If $\mathbf{F}$ is everywhere ________________ to a smooth, oriented surface, $\mathcal{S}$, then $\displaystyle\iint_{\mathcal{S}} \mathbf{F} \bullet d\mathbf{S} = 0$.

The correct words to fill in the blanks are:

A. tangent/tangent.
B. tangent/perpendicular.
C. perpendicular/tangent.
D. perpendicular/perpendicular.

Question 3

Let $\mathcal{S}$ be the cylinder defined by the equation $x^2 + y^2 = 4$, and $-1 \leq z \leq 1$, along with the top and bottom (think of $\mathcal{S}$ as a sealed soup can), parameterized so that the normal vector to $\mathcal{S}$ always points inward around the surface. Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Then the flux of the curl of $\mathbf{F}$ through $\mathcal{S}$ is

A. positive.

B. negative.

C. zero.

D. cannot tell.

Question 4

For $\mathbf{F}$ a C^1-vector field defined on a closed, bounded, oriented C^1-surface $\mathcal{S}$, Stokes Theorem says

A. The vector line integral of $\mathbf{F}$ over $\partial\mathcal{S}$ equals the vector surface integral of $\mathbf{F}$ over $\mathcal{S}$.
B. The circulation of $\mathbf{F}$ around $\partial\mathcal{S}$ equals the flux of $\mathbf{F}$ through $\mathcal{S}$.
C. The work done by $\mathbf{F}$ along $\partial\mathcal{S}$ equals the vector surface integral of $\mathbf{curl}(\mathbf{F})$ over $\mathcal{S}$.
D. The scalar line integral of $\mathbf{F}$ over $\partial\mathcal{S}$ equals the scalar surface integral of $\mathbf{curl}(\mathbf{F})$ over $\mathcal{S}$.
E. None of the above.
F. Nothing! Theorems cannot talk.