Lecture Questions I: F19.AS.110.202 Calculus III

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September 25, 2019

Question 1

How big is the space of solutions to the equation $2x - y = 2$ in $\mathbb{R}^5$?

A. 1-dimensional, a line.
B. 2-dimensional, a plane.
C. 3-dimensional.
D. 4-dimensional.
E. cannot tell.

Question 2

In $\mathbb{R}^3$, the line passing through the origin and the point $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has a parameterization:

A. $x = t - 1$, $y = t - 2$, $z = t - 3$.
B. $x = 1 - t$, $y = 2 - t$, $z = 3 - t$.
C. $x = 1 - (t - 1)$, $y = 2 - (t - 1)$, $z = 3 - (t - 1)$.
D. $x = t - 1$, $y = 2t - 2$, $z = 3t - 3$.

Question 3

A vertical slice (also called a section) through the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = -x^2 - y^2$ in the xyz-space $\mathbb{R}^3$ at $x = -1$ has this graph in the yz-plane at $x = -1$:

A. A circle of radius 1 centered at the origin.

B. A parabola symmetric with respect to the z-axis and opening down with vertical intercept at -1.

C. A parabola symmetric with respect to the z-axis and opening up with vertical intercept at 1.

D. A hyperbola crossing the y-axis at the points $y = 1$ and $y = -1$.

Question 4

Let $\mathbf{f}:\mathbb{R}^4 \to \mathbb{R}^6$, $\mathbf{g}:\mathbb{R}^3 \to \mathbb{R}^4$, and $\mathbf{h}:\mathbb{R}^6 \to \mathbb{R}^7$ be functions which are all differentiable everywhere. Then $D\left(\mathbf{h}\circ\mathbf{f}\circ\mathbf{g}\right)(\mathbf{x})$ is a

A. 3×4-matrix,
B. 6×4-matrix,
C. 7×3-matrix,
D. 6×3-matrix,
E. 4×7-matrix.

Question 5

Given the C^1 functions $\mathbf{f} : V \subset \mathbb{R}^n \to \mathbb{R}^m$ and $\mathbf{g} : U \subset \mathbb{R}^p \to \mathbb{R}^q$, if $\mathbf{h}(\mathbf{x}) = \frac{\mathbf{f}(\mathbf{x})}{\mathbf{g}(\mathbf{x})}$ makes sense, we can calculate $D\mathbf{h}(\mathbf{x}_0)$ via the Quotient Rule for differentiation. Which one of the following facts about the Quotient Rule is NOT true:

A. It is only valid for real-valued functions (Hence $m = q = 1$).
B. The form is the same as in SVC even though the derivatives are matrices.
C. The denominator function can be mapped to the origin at $\mathbf{x}_0$ as long as the numerator function also maps to the origin there (so that a limit exists at $\mathbf{x}_0$).
D. The domain of $\mathbf{h}$ is the intersection of V and U (Hence $n = p$).

Question 6

True or False: It is easy to visually check whether a parameterized curve in $\mathbb{R}^n$ is differentiable by checking for corners or endpoints in its image.

A. True.
B. False.
C. Not sure. The coin landed on its edge.