Lecture Questions I: F19.AS.110.202 Calculus III

Richard Brown, Director of Undergraduate Studies

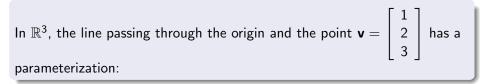
Mathematics Department

September 25, 2019

Richard Brown (Mathematics Department) Lecture Questions I: F19.AS.110.202 Calculus

How big is the space of solutions to the equation 2x - y = 2 in \mathbb{R}^5 ?

- 1-dimensional, a line.
- 2-dimensional, a plane.
- 3-dimensional.
- 4-dimensional.
- cannot tell.



A vertical slice (also called a section) through the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = -x^2 - y^2$ in the *xyz*-space \mathbb{R}^3 at x = -1 has this graph in the *yz*-plane at x = -1:

- A circle of radius 1 centered at the origin.
- A parabola symmetric with respect to the *z*-axis and opening down with vertical intercept at −1.
- A parabola symmetric with respect to the z-axis and opening up with vertical intercept at 1.
- O A hyperbola crossing the y-axis at the points y = 1 and y = -1.

Let $\mathbf{f} : \mathbb{R}^4 \to \mathbb{R}^6$, $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^4$, and $\mathbf{h} : \mathbb{R}^6 \to \mathbb{R}^7$ be functions which are all differentiable everywhere. Then $D(\mathbf{h} \circ \mathbf{f} \circ \mathbf{g})(\mathbf{x})$ is a

- 3×4 -matrix,
- 6 × 4-matrix,
- 7 × 3-matrix,
- 6 × 3-matrix,
- $4 \times 7 \text{-matrix.}$

Given the C^1 functions $\mathbf{f}: V \subset \mathbb{R}^n \to \mathbb{R}^m$ and $\mathbf{g}: U \subset \mathbb{R}^p \to \mathbb{R}^q$, if $\mathbf{h}(\mathbf{x}) = \frac{\mathbf{f}(\mathbf{x})}{\mathbf{g}(\mathbf{x})}$ makes sense, we can calculate $D\mathbf{h}(\mathbf{x}_0)$ via the Quotient Rule for differentiation. Which one of the following facts about the Quotient Rule is NOT true:

- It is only valid for real-valued functions (Hence m = q = 1).
- The form is the same as in SVC even though the derivatives are matrices.
- The denominator function can be mapped to the origin at x₀ as long as the numerator function also maps to the origin there (so that a limit exists at x₀).
- **(2)** The domain of **h** is the intersection of V and U (Hence n = p).

True or False: It is easy to visually check whether a parameterized curve in \mathbb{R}^n is differentiable by checking for corners or endpoints in its image.

- True.
- False.
- On the sure. The coin landed on its edge.