Math 302: Ordinary Differential Equations
Extra 2nd Order ODE problem: Resonance

A car supported by a MacPherson strut (shock absorber system) travels on a bumpy road at a constant velocity *v*. The equation modeling the motion of the car is

$$80\ddot{x} + 10000x = 2500\cos\left(\frac{\pi v t}{6}\right),$$

where *x* represents the vertical position of the car's axle relative to its equilibrium position, and the basic units of measurement are feet and feet per second (this is actually just an example of a forced, un-damped harmonic oscillator, if that is any help).

The constant numbers above are related to the characteristics of the car and the strut. Note that the coefficient of time *t* (inside the cosine) in the forcing term on the right hand side is a frequency, which in this case is directly proportional to the velocity *v*.

(a) Find the general solution to this non-homogeneous ODE. Note that your answer will have a term in it which is a function of *v*.
(b) Determine the value of *v* for which the solution is undefined (you should present your final answer in miles per hour, as opposed to feet per second).
(c) For a set of initial values $x(0) = 0$, $\dot{x}(0) = 0$, graph the solutions for a few values of *v* near your answer in part b and not so near. Discuss the differences in these graphs and the importance of the special value of *v* in part b. (Hint: This special value of *v* induces what is called resonance in the car).
(d) Write the IVP (the ODE with the initial conditions in part (c)) as a non-homogeneous first order system of ODEs. We will learn how to solve such a system in time.