

Math 302: Ordinary Differential Equations
Extra bifurcation problem
September 21, 2012
Due: September 28, 2012

Consider the population model for a species of fish in a lake

$$\frac{dP}{dt} = \frac{-P^2}{50} + 2P,$$

where P is measured in thousands of fish and t is measured in years. The US Fish and Wildlife Service, which is managing the lake, wants to issue fishing licenses for the harvesting of the fish (this amounts to a constant term being subtracted off of the right hand side above, which is a function of h , the number of licenses issued). Each fishing license is valid for the annual take of 3000 fish.

Draw a bifurcation diagram for the above ODE with the added parameter part, and answer the following questions.

- (a) What is the largest number of licenses that can be issued if the goal is to keep a stable population of fish in the lake over the long term?
- (b) If the largest number of licenses is actually issued, what is the expected long term stable population of fish in the lake?
- (c) Solve the IVP given by the above differential equation and the initial value $P(0) = 2$ (This corresponds to an initial population of 2000 fish in the lake, and an assumption that there will be no harvesting, $h = 0$).
- (d) As an expert consultant to the USFWS, discuss the ramifications of issuing the maximal number of licenses allowed by a mathematical model in the presence of real world issues which may temporary affect populations (drought, flooding, unlawful fishing, pollution, etc.)
- (e) What is your final recommendation, in terms of the number of licenses that should be issued, to the USFWS?