HOMEWORK PROBLEM SET 11: DUE NOVEMBER 16, 2018

110.302 DIFFERENTIAL EQUATIONS
PROFESSOR RICHARD BROWN

Question 1. For each system, solve for the general solution and determine a fundamental matrix for the system. Then also determine the fundamental matrix $\Phi(t)$, where $\Phi(0) = I_2$.

(a) $\mathbf{x}' = \begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix} \mathbf{x}$.

(b) $\mathbf{x}' = \begin{bmatrix} -4 & 2 \\ -1 & -1 \end{bmatrix} \mathbf{x}$.

(c) $\mathbf{x}' = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} \mathbf{x}$.

(d) $\mathbf{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} \mathbf{x}$.

Question 2. Solve the IVP

$$\mathbf{x}' = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

by using the fundamental matrix $\Phi(t)$, where $\Phi(0) = I_2$.

Question 3. For the following systems, find a general solution, draw a direction field and plot enough trajectories to fully characterize the nature of the solutions to the system.

(a) $\mathbf{x}' = \begin{bmatrix} -\frac{3}{2} & 1 \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix} \mathbf{x}$.

(b) $\mathbf{x}' = \begin{bmatrix} -4 & 8 \\ -2 & 4 \end{bmatrix} \mathbf{x}$.

Question 4. Solve the IVP

$$\mathbf{x}' = \frac{1}{2} \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

and draw the solution in the phase plane. Then also graph each component of the solution as functions of t. Feel free to use technology when graphing.

Question 5. Show that all solutions to

$$\mathbf{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{x}$$

approach the origin as $t \to \infty$ if and only if $a + d < 0$ and $ad - bc > 0$.