Lecture Questions II: 110.302 Differential Equations

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Question 1

In the Variation of Parameters Method for finding a particular solution to a nonhomogeneous, second-order linear ODE $L[y] = g(t)$, determine which one of the following is a true statement:

A. The method only works when $g(t)$ has a certain type (a sum of products of exponentials, sines or cosines and polynomials.)

B. The resulting system in the method,
$$u_1' y_1 + u_2' y_2 = 0, \quad u_1' y_1' + u_2' y_2' = g(t),$$
where $y_1(t)$ and $y_2(t)$ comprise the fundamental set of homogeneous solutions, may be inconsistent. In this case, $Y(t) = u_1 y_1 + u_2 y_2$ would not exist.

C. It is possible for one of the two of $u_1(t)$ and $u_2(t)$ to be a constant as long as the other is not.

D. One must check the Wronskian to see if $Y(t)$ is actually independent of y_1 and y_2.

E. None of the previous statements is true.

Question 2

Let

$$ay'' + by' + cy = 0, \quad y(t_0) = y_0, \quad y'(t_0) = y_0'$$

be a homogeneous, second-order, linear IVP with constant coefficients. If $a, b, c > 0$, what can we say about the long term behavior of the solutions?

A. Solutions exist and are uniquely defined for all $t \in \mathbb{R}$, and tend to either ∞ or $-\infty$ as $t \to \infty$.

B. Solutions exist and are uniquely defined for all $t \in \mathbb{R}$, and all tend to 0 as $t \to \infty$.

C. Solutions exist and are uniquely defined only for certain choices of a, b, and c, and where these solutions tend to depends on the initial data.

D. Solutions exist and are uniquely defined always, but there is not enough information to determine where solutions go.

E. Not enough information available to determine whether solutions exist and/or are uniquely defined.

Question 3

Determine the truth of the following two statements about the Wronskian determinant of any two solutions to $y'' + p(t)y' + q(t)y = 0$:

(1) The Wronskian can be used to determine precisely where solutions to the ODE exist and are unique.

(2) If one solution is already known, one can use the Wronskian determinant to construct another independent solution.

A. Both are true.

B. (1) is true and (2) is false.

C. (1) is false and (2) is true.

D. Both are false.