

CHALLENGE PROBLEM SET: WEEK 8

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Show that $W(5, \sin^2 t, \cos 2t) = 0$, for all t . Now show that the three functions are linearly dependent by finding a linear relation between them, without using the Wronskian.

Question 2. Use the method of Undetermined Coefficients to find the general solution to

$$y^{(4)} - 2y'' + y = e^t + \sin t.$$

Question 3. Show that the general solution to $y^{(4)} - y = 0$ can be written

$$y(t) = c_1 \cos t + c_2 \sin t + c_3 \sinh t + c_4 \cosh t.$$

Determine the solution satisfying the initial conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$, and $y'''(0) = 1$. Why is it convenient to use the solutions $\sinh t$ and $\cosh t$ instead of e^t and e^{-t} ?

Question 4. Consider the linear nonhomogeneous system

$$x' = p_{11}(t)x + p_{12}(t)y + g_1(t),$$

$$y' = p_{21}(t)x + p_{22}(t)y + g_2(t).$$

Show that if $x = x_1(t), y = y_1(t)$ and $x = x_2(t), y = y_2(t)$ are two solutions to this system, then $x = x_1(t) - x_2(t), y = y_1(t) - y_2(t)$ is a solution to the corresponding homogeneous system. This is similar to what you saw in nonhomogeneous linear ODEs of higher order.

Question 5. This week's homework assignment included a problem to show that if y_1 is a solution to

$$y''' + p_1(t)y'' + p_2(t)y' + p_3(t)y = 0,$$

then the substitution $y = y_1 v(t)$ leads to the following second order equation for v' :

$$y_1 v''' + (3y_1' + p_1 y_1) v'' + (3y_1'' + 2p_1 y_1' + p_2 y_1) v' = 0.$$

Use this form of Reduction of Order to solve the differential equation

$$(2-t)y''' + (2t-3)y'' - ty' + y = 0, \quad t < 2; \quad y_1(t) = e^t, \quad y_2(t) = te^t.$$

Question 6. Verify that the given matrix satisfies the matrix differential equation:

$$\Psi' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \Psi, \quad \Psi(t) = \begin{bmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{bmatrix}.$$