CHALLENGE PROBLEM SET: WEEK 5

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Solve the following:

(a) $\dfrac{2ty}{t^2+1} - 2t - \left(2 - \ln(t^2+1)\right) y' = 0, \quad y(5) = 0.$

(b) $x^2y + 2xy^2 \dfrac{dy}{dx} = 0$. Note: This ODE is not exact.

(c) $2xy^{-1} + 2ye^{2x} - 1 + \left(e^{2x} + y^2 - x^2y^{-2}\right) y' = 0.$

Question 2. For the following, verify that the ODE is not exact. In each, an integrating factor is given but only up to some unknown constant(s). Find an the appropriate value for each of the constants that will render each ODE exact. Then solve.

(a) $(3e^x y + x)\, dx + e^x dy = 0, \quad y(0) = 1, \quad \mu(x,y) = e^{rx}.$

(b) $\left(5xy^2 - 2y\right) dx + \left(3x^2y - x\right) dy = 0, \quad \mu(x,y) = x^a y^b.$

Question 3. The ODE $y' = e^{2x} + y - 1$ is also not exact. Find an integrating factor that makes it exact, and use it to solve the differential equation. Now solve it using the methods of Section 2.1, knowing that it is also linear.

Question 4. Use the method of successive approximations (see either the Existence and Uniqueness worksheet or the book), with $\varphi_0(t) = 0$, to solve the IVP

$$y' = -y - 1, \quad y(0) = 0,$$

by doing the following:

(a) Determine the form of $\varphi_n(t)$ for arbitrary choice of n.

(b) Determine a functional form for $\varphi(t) = \lim\limits_{n\to\infty} \varphi_n(t)$.

(c) Show that $\varphi(t)$ solves the IVP.

(d) Verify that the solution you found is the same one you would find by solving the IVP using separation of variables.

Question 5. Do the exercises of the Existence and Uniqueness worksheet.