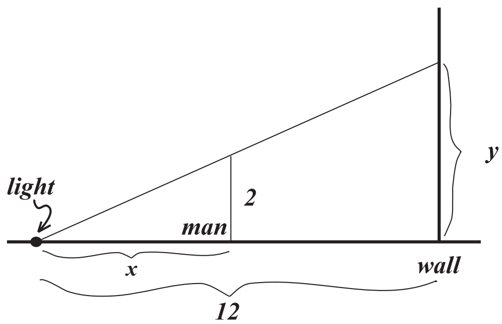
EXAMPLE: PROBLEM 3.9.16 RELATED RATES

110.109 CALCULUS I (PHYS SCI & ENG)
PROFESSOR RICHARD BROWN

Question 1. *A spotlight on the ground shines on a wall 12 meters away. If a two meter man walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the wall decreasing when he is 4 meters from the wall?*

Strategy: The ultimate question here involves the height of the man's shadow, which will be diminishing as he walks away from the light. The light, the floor and the wall form right triangles with respect to the man and his shadow. We solve this problem using as our relation similar triangles. Differentiating the similar triangles equation with respect to time, we form the related rates equation, and solve the problem this way.

Solution: First, we draw a picture of the situation so that we can name the quantities we will need for this problem. In the problem, if the light is at the lower left corner of the triangle, then the man's distance from the light, in meters, will be called x. We will call the height of his shadow y, also in meters. The man forms a right triangle with respect to the floor and the light. His shadow will also form a right triangle with respect to the floor and the light, and the angle at the light is precisely the same. The concept of similar triangles says, that the corresponding sides of two triangle with the same angles satisfy the same ratios (the trig functions of the angles are a way to show that this is true. FOr our purposes, the small triangle (with the man as height), and the big triangle (with height along the wall) are similar, and hence the ratio of adjacent side length to opposite side length (with respect to the light) will be the same for both. This translates to



$$\frac{\text{adjacent side length}}{\text{opposite side length}} = \frac{x}{2} = \frac{12}{y},$$

which implies that $xy = 24$ is an equation relating x and y.

Differentiating this last equation with respect to t (both x and y are changing with respect to time if the man is walking toward the wall, no?), we get

$$\frac{d}{dt}\left[xy = 24\right], \quad \Longleftrightarrow \quad \frac{dx}{dt}y + x\frac{dy}{dt} = 0, \quad \Longleftrightarrow \quad \frac{dy}{dt} = -\frac{y}{x}\frac{dx}{dt}.$$

Date: November 17, 2011.

Hence, at the moment when the man is 4 meters from the building, we have $x = 8$, and $\frac{dx}{dt} = 1.6$. Using the relation of similar triangles, we can also find y when $x = 8$, and this is $8y = 24$, or $y = 3$. Hence

$$\frac{dy}{dt} = -\frac{y}{x}\frac{dx}{dt} = -\frac{3}{8}(1.6) = -\frac{4.8}{8} = -.6 \text{ meters per second.}$$

Note that this quantity is negative, suggesting that the value of y is decreasing. This makes sense as the man walks toward to wall and away from the light, no?