EXAMPLE: PROBLEM 3.4.54: THE CHAIN RULE

110.109 CALCULUS I (PHYS SCI & ENG) PROFESSOR RICHARD BROWN

Question. Find the equation of the line tangent to $f(x) = \sin x + \sin^2 x$ at the point (0,0).

Strategy: With the point in question $(x_0, y_0) = (0, 0)$, we calculate the derivative of f(x) at x = 0 and construct the line via the equation $y - y_0 = f'(0)(x - x_0)$.

Solution: f(x) is certainly differentiable at x = 0, and using the Sum Rule and the Chain Rule on the second summand (the outside function is x^2 and the inside function is $\sin x$), we get

$$f'(0) = \frac{d}{dx} \left[\sin x + \sin^2 x \right] \Big|_{x=0} = \left(\cos x + 2\sin x \cos x \right) \Big|_{x=0} = \cos 0 + 2\sin 0 \cos 0 = 1.$$

Hence the equation of the tangent line is

$$y - y_0 = f'(0)(x - x_0)$$

 $y - 0 = 1(x - 0), \text{ or } y = x$

The graph of f(x) near x = 0, along with the tangent line at (0,0) is here:

