EXAMPLE: PROBLEM 2.3.9 LIMIT LAWS

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Question 1. Find the limit using the appropriate Limit Laws, stating the laws used, of $\lim_{x\to 2} \sqrt{\frac{2x^2+1}{3x+2}}$

Strategy: We will slowly work our way into the expression by employing the limit laws to get to the point where the actual limits. If at any point, we find a violation in the use of the limit laws, we must stop since the limit cannot then be calculated in this fashion.

Solution: use the Root and Quotient Laws, again noting that the Quotient Law will only ultimately work only if the individual limits exist and the denominator does not limit to 0:

$$\lim_{x \to 2} \sqrt{\frac{2x^2 + 1}{3x + 2}} = \sqrt{\lim_{x \to 2} \left(\frac{2x^2 + 1}{3x + 2}\right)} = \sqrt{\frac{\lim_{x \to 2} \left(2x^2 + 1\right)}{\lim_{x \to 2} \left(3x + 2\right)}}.$$

Now we use the Sum, Power and Constant Multiple Laws:

$$\sqrt{\frac{\lim_{x \to 2} (2x^2 + 1)}{\lim_{x \to 2} (3x + 2)}} = \sqrt{\frac{\lim_{x \to 2} (2x^2) + \lim_{x \to 2} 1}{\lim_{x \to 2} (3x) + \lim_{x \to 2} 2}}$$
$$= \sqrt{\frac{2\left(\lim_{x \to 2} x\right)^2 + \left(\lim_{x \to 2} 1\right)}{3\left(\lim_{x \to 2} x\right) + \left(\lim_{x \to 2} 2\right)}}$$

Now since we know that $\lim_{x \to a} c = c$ and $\lim_{x \to a} x = a$, we get that all of our limit laws are fine, and

$$\sqrt{\frac{2\left(\lim_{x \to 2} x\right)^2 + \left(\lim_{x \to 2} 1\right)}{3\left(\lim_{x \to 2} x\right) + \left(\lim_{x \to 2} 2\right)}} = \sqrt{\frac{2\left(2\right)^2 + \left(1\right)}{3\left(2\right) + \left(2\right)}} = \sqrt{\frac{9}{8}} = \frac{3}{\sqrt{8}}$$
$$\lim_{x \to 2} \sqrt{\frac{2x^2 + 1}{2x + 2}} = \sqrt{\frac{9}{8}}.$$

Hence

$$\lim_{x \to 2} \sqrt{\frac{2x^2 + 1}{3x + 2}} = \sqrt{\frac{9}{8}}$$

Date: September 26, 2011.