EXAMPLE: PROBLEM 2.3.28: LIMIT LAWS

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Question 1. *Find* $\lim_{h\to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$*, if it exists.*

Strategy: An attempt at using the Quotient Limit Law will not work here, since the limit of the denominator, as h goes to 0, is 0 (this would be like a direct substitution of $h = 0$ into the ratio, giving you a 0 over 0 indeterminate form. Instead, we will algebraic manipulation on the quotient to try to "see" the limit via the Limit Laws.

Solution: First, note that

$$(3+h)^{-1} - 3^{-1} = \frac{1}{3+h} - \frac{1}{3} = \frac{3}{3(3+h)} - \frac{3+h}{3(3+h)} = \frac{3-3-h}{3(3+h)} = \frac{h}{3(3+h)}.$$

Hence

$$\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} = \lim_{h\to 0} \frac{\frac{h}{3(3+h)}}{h} = \lim_{h\to 0} \frac{1}{3(3+h)}.$$

Using the Quotient Limit Law (which works as long as the numerator and denominator both have limits and the denominator does not limit to 0), as well as the Constant Multiple and the Sum Limit Laws, we get

$$\lim_{h\to 0} \frac{1}{3(3+h)} = \frac{\lim_{h\to 0} 1}{\lim_{h\to 0} 3(3+h)} = \frac{1}{3 \lim_{h\to 0}(3+h)} = \frac{1}{3\left(\left(\lim_{h\to 0} 3\right) + \left(\lim_{h\to 0} h\right)\right)} = \frac{1}{9}.$$

Hence $\lim_{h\to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h} = \dfrac{1}{9}$.

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