

110.106 CALCULUS I: BIOLOGICAL AND SOCIAL SCIENCES

Fall 2009

A Mean Value/Rolle's Theorem Example

Exercise 5.1.46) Assume that $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) . Show that if $f(a) < f(b)$, then f' is positive at some point between a and b .

Solution: This is a Mean Value Theorem (MVT) problem. The solution is found by showing that the premises of the MVT hold here for $f(x)$, so the conclusion holds also. And the conclusion contains the solution to this problem.

Indeed, since $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) , by the MVT, there exists at least one number $c \in (a, b)$ where

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

But since $f(a) < f(b)$ by premise above, we know the left hand side of this last equation is positive (that is, $a < b$ also). Thus the right hand side is also positive, and hence f' is positive at some point (namely the c) between a and b .