Homework #8 Due W/D 11/2

Section 4.8

6. $f(x) = \sin x$ $f(a) = \sin(0) = 0$ $f(x) \approx f(a) + f'(a)(x-a)$

$f'(x) = \cos x$ $f'(a) = \cos(0) = 1$ $f(0.01) \approx 0 + 1(0.01 - 0)$

$x = 0.01$ $f(0.01) \approx 0.01$

$a = 0$ $f(0.01) = \sin(0.01) = .00995$

14. $f(x) = \frac{1}{1-x}$, $a = 2$ $L(x) = f(a) + f'(a)(x-a)$

$f'(x) = -(1-x)^{-2}(-1)$ $L(x) = -1 + 1(x-2)$

$= \frac{1}{(1-x)^2}$

$f'(2) = \frac{1}{(1-2)^2} = \frac{1}{(-1)^2} = 1$

$f(2) = \frac{1}{1-2} = \frac{1}{-1} = -1$

$\left.\begin{array}{l} L(x) = -1 + (x-2) \\ L(x) = x - 3 \end{array}\right\}$ Either is okay

18. $f(x) = \ln(1+2x)$, $a = 0$ $L(x) = f(a) + f'(a)(x-a)$

$f'(x) = \frac{2}{1+2x}$ $L(x) = 0 + 2(x-0)$

$f(0) = \ln(1+2(0)) = \ln(1+0) = \ln(1) = 0$ $L(x) = 2x$

$f'(0) = \frac{2}{1+2(0)} = \frac{2}{1} = 2$

26. $f(x) = e^{2x+1}$, $a = -1/2$ $L(x) = f(a) + f'(a)(x-a)$

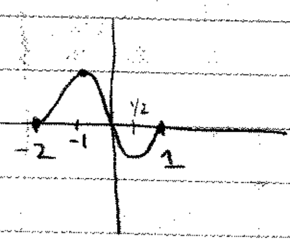
$f'(x) = 2e^{2x+1}$ $L(x) = 1 + 2(x - (-1/2))$

$f(-1/2) = e^{2(-1/2)+1} = e^{-1+1} = e^0 = 1$

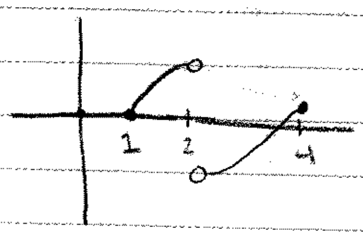
$f'(-1/2) = 2e^{2(-1/2)+1} = 2e^{-1+1} = 2e^0 = 2$

$\left.\begin{array}{l} L(x) = 1 + (2x+1) \\ L(x) = 2x + 2 \end{array}\right\}$ Either is okay

Section 5.1

10. 

Global Maximum and Global Minimum in the interior of $[-2, 1]$. } Answers May vary

12. 

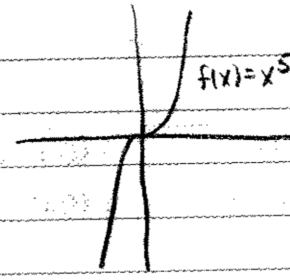
Continuous on $[1, 4]$ except at $x=2$, but has no global minimum or global maximum in its domain.

* Answers May Vary

24. $f(x) = x^5$

$f'(x) = 5x^4$

$f'(0) = 5(0)^4 = 5(0) = 0$

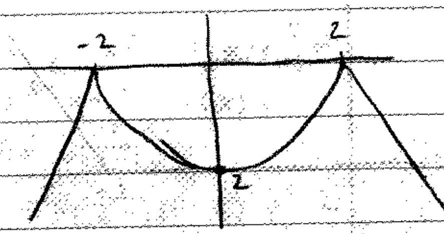


(Need not Graph)

$\therefore$ f(x) is not a local extrema

since if $x<0,\ f(x) < 0 = f(0)$ and if $x>0,\ f(x) > 0 = f(0)$.

30. $f(x) = -|x^2-4| = \begin{cases} -(x^2-4) & |x| \geq 2 \\ x^2-4 & |x| < 2 \end{cases}$



$f(2) = f(-2) = 0$ and since absolute values are always nonnegative our f(x) is always 'nonpositive'. So, $x=2$, $x=-2$ are local maxima

Now consider $x=-2$:

$\lim_{h\to0^-} \frac{f(-2+h)-f(-2)}{h} = \lim_{h\to0^-} \frac{-((-2+h)^2-4)-0}{h} = \lim_{h\to0^-} \frac{-(4-4h+h^2-4)-0}{h}$

$= \lim_{h\to0^-} \frac{h(4-h)}{h} = \lim_{h\to0^-} 4-h = 4-0 = 4$

$\lim_{h\to0^+} \frac{f(-2+h)-f(-2)}{h} = \lim_{h\to0^+} \frac{(-2+h)^2-4-0}{h} = \lim_{h\to0^+} \frac{4-4h+h^2-4}{h}$

$= \lim_{h\to0^+} \frac{h(h-4)}{h} = \lim_{h\to0^+} h-4 = 0-4 = -4$

$\therefore$ f is not differentiable at $x=-2$, since $\lim_{h\to0^-} \neq \lim_{h\to0^+}$

Also, consider $x=2$:

$\lim_{h\to0^-} \frac{f(2+h)-f(2)}{h} = \lim_{h\to0^-} \frac{(2+h)^2-4-0}{h} = \lim_{h\to0^-} \frac{4+4h+h^2-4}{h}$

$= \lim_{h\to0^-} \frac{h(4+h)}{h} = \lim_{h\to0^-} 4+h = 4+0 = 4.$

$\lim_{h\to0^+} \frac{f(2+h)-f(2)}{h} = \lim_{h\to0^+} \frac{-((2+h)^2-4)-0}{h} = \lim_{h\to0^+} \frac{-(4+4h+h^2-4)}{h}$

$= \lim_{h\to0^+} \frac{-h(4+h)}{h} = \lim_{h\to0^+} -(4+h) = -(4+0) = -4.$

$\therefore$ f is not differentiable at $x=2$, since $\lim_{h\to0^-} \neq \lim_{h\to0^+}$

34. $\frac{dN}{dt} = rN(1-\frac{N}{K}),\ t \geq 0.$

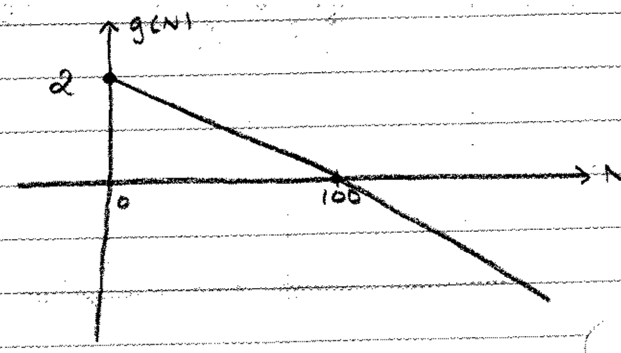
(a) $\frac{dN}{dt} = rN(1-\frac{N}{K})$

$\frac{1}{N}\frac{dN}{dt} = r(1-\frac{N}{K})$

$g(N) = r(1-\frac{N}{K})$

(b) $g(N) = 2(1-\frac{N}{100})$

Graph $\longrightarrow$



Maximal when $N=0$.

36. $f(x) = 1/x$, $x \in [1,2]$

(a) $(1,1), (2, 1/2) \Rightarrow m = \frac{1 - 1/2}{1-2} = \frac{1/2}{-1} = \underline{-1/2}$

(b) Note that f is continuous on $[1,2]$ and differentiable on $(1,2)$. Therefore, by the Mean Value Theorem (MVT), there exists $c \in (1,2)$ s.t. $\frac{f(2) - f(1)}{2-1} = f'(c)$.

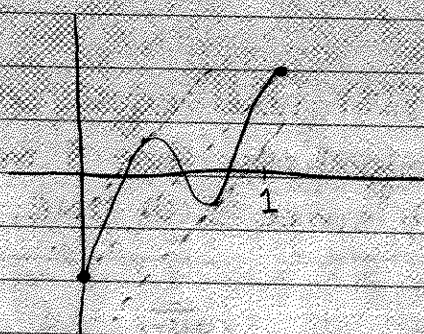
But $\frac{f(2)-f(1)}{2-1} = -1/2$ by part (a). So, we find c s.t. $f'(c) = -1/2$.

$f'(x) = -x^{-2} = -1/x^2$

Then $f'(c) = -1/c^2 = -1/2 \Rightarrow 2 = c^2 \Rightarrow c = \pm\sqrt{2}$

But $-\sqrt{2} \notin [1,2]$. So, $\underline{c = \sqrt{2}}$

44.



An example of a graph that is continuous on $[0,1]$, differentiable on $(0,1)$ and has exactly two points $(c_1, f(c_1))$, $(c_2, f(c_2))$ where $f'(c_1) = f'(c_2) = \frac{f(1)-f(0)}{1-0}$. At least one is ensured by the MVT.

46. __Proposition__: Assume that f is continuous on $[a,b]$ and differentiable on (a,b). If $f(a) < f(b)$, then f' is positive at some point between a and b.

__Proof__: We can apply the MVT so that there exists $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. Clearly $b - a > 0$. Also, since $f(a) < f(b)$, then $f(b) - f(a) > 0$. Therefore, $f'(c) = \frac{f(b)-f(a)}{b-a} > 0$. So f' must be positive at some point between a and b. ∎

48. $s(t) = \frac{1}{10}t^2$, $0 \le t \le 10$

(a) Average Velocity $= \frac{s(10) - s(0)}{10 - 0} = \frac{10 - 0}{10 - 0} = \frac{10}{10} = 1$.

(b) Instantaneous Velocity $= s'(t) = \frac{1}{10}(2t) = t/5$

(c) Avg V = Inst V $\Rightarrow 1 = t/5 \Rightarrow \underline{t = 5}$

54. $f(x) = e^{-|x|}$ $\quad x \in [-2, 2]$

(a) $f(-2) = e^{-|-2|} = e^{-2}$

$f(2) = e^{-|2|} = e^{-2}$ $\quad \therefore f(-2) = f(2)$.

(b) $x > 0 \Rightarrow f(x) = e^{-x} \Rightarrow f'(x) = -e^{-x}$

$x < 0 \Rightarrow f(x) = e^{-(-x)} = e^{x} \Rightarrow f'(x) = e^{x}$

Not defined at zero since $|x|$ as a corner point

and is not differentiable at $x=0$.

↳ Also, $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{e^{-|h|} - e^{-|0|}}{h} = \lim_{h \to 0^-} \frac{e^{h} - 1}{h} = 1$

$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{e^{-|h|} - e^{-|0|}}{h} = \lim_{h \to 0^+} \frac{e^{-h} - 1}{h} = -1$

So, $f'(0)$ DNE.

(c) $[-2, 0) \Rightarrow f'(x) = -e^{-x} < 0$.

$(0, 2] \Rightarrow f'(x) = e^{x} > 0$.

$f'(0)$ DNE. Thus, $f'(x)$ never equals zero.

(d) f does not satisfy the hypothesis that f must be differentiable on $(-2, 2)$, since $f'(0)$ DNE.

(e)