Homework #7 *Answers need not be reduced* Due W/D 10/26

Section 4.6

14. $f(x)=\frac{x-e^{-x}}{1+xe^{-x}}$ $f'(x)=\frac{\frac{d}{dx}(x-e^{-x})(1+xe^{-x})-\frac{d}{dx}(1+xe^{-x})(x-e^{-x})}{(1+xe^{-x})^2}$

$$=\frac{(1+e^{x})(1+xe^{-x})-(e^{-x}-xe^{-x})(x-e^{-x})}{(1+xe^{-x})^2}$$

28. $g(s)=\exp(\tan(s^3))$ $g'(s)=\frac{d}{ds}(\tan(s^3))\ \exp(\tan(s^3))$

$$=\sec^2(s^3)\,\frac{d}{ds}(s^3)\ \exp(\tan(s^3))$$
$$=3s^2\sec^2(s^3)\exp(\tan(s^3))$$

GRADE

40. $f(x)=3^{x^3-1}$ $f'(x)=\frac{d}{dx}(x^3-1)\ (\ln 3)\ 3^{x^3-1}$

$$=3x^2(\ln(3))(3^{x^3-1})$$

48. $h(t)=6^{\sqrt{6t^6-6}}$ $h'(t)=\frac{d}{dt}(\sqrt{6t^6-6})(\ln 6)\ 6^{\sqrt{6t^6-6}}$

$$=\left(\tfrac{1}{2}(6t^6-6)^{-1/2}\ \tfrac{d}{dt}(6t^6-6)\right)(\ln 6)\ 6^{\sqrt{6t^6-6}}$$
$$=\tfrac{1}{2}(6t^6-6)^{-1/2}(36t^5)(\ln 6)\ 6^{\sqrt{6t^6-6}}$$

54. $\lim_{h\to 0}\frac{e^{5h}-1}{3h}=\frac{5}{3}\lim_{h\to 0}\frac{e^{5h}-1}{5h}$. Let $l=5h$. Then as $h\to 0$, $l\to 0$ as well. So, $\lim_{h\to 0}\frac{e^{5h}-1}{3h}=\frac{5}{3}\lim_{l\to 0}\frac{e^{l}-1}{l}=\frac{5}{3}(1)=\frac{5}{3}$.

60. $N(t)=N_0e^{rt}$, $t\geq 0$ $N_0>0$ constant, $r\in\mathbb{R}$

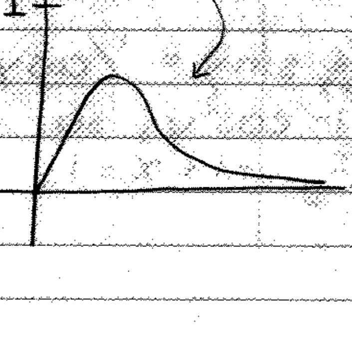
(a) $N(0)=N_0e^{r(0)}=N_0e^0=N_0$.

(b) $\frac{dN}{dt}=\frac{d}{dt}(N_0e^{rt})=rN_0e^{rt}=r(N_0e^{rt})=rN$.

GRADE

64. $R(P)=\alpha Pe^{-\beta P}$, $P\geq 0$, $\alpha,\beta>0$ constants

(a) $R(P)=2Pe^{-P}$



(b) $R'(P)=\frac{d}{dP}(\alpha P)e^{-\beta P}+\frac{d}{dP}(e^{-\beta P})(\alpha P)$

$$=\alpha e^{-\beta P}-\alpha\beta Pe^{-\beta P}$$

(c) $0=e^{-\beta P}(\alpha-\alpha\beta P)$

$\cancel{0=e^{-\beta P}}$ No Solutions

$0=\alpha-\alpha\beta P$

$\alpha\beta P=\alpha$

$P=1/\beta$

Section 4.7

2. $f(x) = \sqrt{x-1}, \quad x \geq 1.$

$y = \sqrt{x-1}$

$x = \sqrt{y-1} \qquad \therefore f^{-1}(x) = x^2+1$

$x^2 = y-1$

$y = x^2+1$

(i) $[f^{-1}(x)]' = 2x$

(ii) $f'(x) = \frac{1}{2}(x-1)^{-1/2}$

$[f^{-1}(x)]' = 1/f'(x^2+1) = 1/\frac{1}{2}(x^2+1-1)^{-1/2} = 2(x^2)^{1/2} = 2x.$

8. $f(x) = -x^2+3, \quad x \geq 0$

$y = -x^2+3$

$x = -y^2+3$

$x-3 = -y^2$

$y^2 = 3-x$

$y = \sqrt{3-x}$

$f^{-1}(x) = \sqrt{3-x}$

$f(2) = -1 \Rightarrow f^{-1}(-1) = 2$

$f'(x) = -2x$

$\frac{d}{dx}[f^{-1}(-1)] = 1/f'(f^{-1}(-1))$

$= 1/f'(2) = 1/-2(2) = -1/4.$

GRADE 18. $f(x) = \tan x \quad 0 < x < \pi/2$ $\qquad \sqrt{3} = \tan x \Rightarrow x = \pi/3 \Rightarrow f^{-1}(\sqrt{3}) = \pi/3$

$f'(x) = \sec^2 x \qquad f^{-1}(x) = \arctan x.$

$\frac{d}{dx}[f^{-1}(\sqrt{3})] = 1/f'(f^{-1}(\sqrt{3})) = 1/f'(\pi/3) = 1/\sec^2(\pi/3) = \cos^2(\pi/3)$

$= (1/2)^2 = 1/4.$

$\boxed{\text{OR}}$

$\frac{d}{dx}\arctan(\sqrt{3}) = 1/1+(\sqrt{3})^2 = 1/1+3 = 1/4$

22. $y = \arcsin x, \quad -1 \leq x \leq 1$ $\quad$ inverse of $y = \sin x$ ← Notation of Text

$-\pi/2 \leq x \leq \pi/2.$

$\frac{d}{dx}\sin x = \cos x$

So, $\frac{d}{dx}\arcsin x = \frac{1}{dx/dy} = 1/\frac{d}{dy}\sin y = 1/\cos y$

$x = \sin y \Rightarrow x^2 = \sin^2 y = 1-\cos^2 y$, so $\cos y = \sqrt{1-x^2}.$

Therefore, $\frac{d}{dx}\arcsin x = 1/\sqrt{1-x^2}, \quad -1 \leq x \leq 1.$

29. $f(x) = \ln(x^3) = 3\ln x$

$f'(x) = \frac{d/dx\,(x^3)}{x^3} = \frac{3x^2}{x^3} = \frac{3}{x}$

32. $f(x) = (\ln x)^3$

$f'(x) = 3(\ln x)^2 \frac{d}{dx}(\ln x)$

$= 3(\ln x)^2 (1/x)$

36. $f(x) = \ln\sqrt{2x^2 - x}$

$f'(x) = \frac{d}{dx}(\sqrt{2x^2-x}\,)\left(1/\sqrt{2x^2-x}\right)$

$= (\frac{1}{2}(2x^2-x)^{-1/2}\,\frac{d}{dx}(2x^2-x)\,\left(1/\sqrt{2x^2-x}\right)$

$= \frac{1}{2}(2x^2-x)^{-1/2}\,(4x-1)\left(1/\sqrt{2x^2-x}\right)$

43. $f(x) = \ln(\sin x)$

$f'(x) = \frac{d}{dx}(\sin x)\ (1/\sin x)$

$= \cos x\ (1/\sin x) = \cot x$

48. $f(x) = x^2 \ln x^2$

$f'(x) = \frac{d}{dx}(x^2)\ln x^2 + \frac{d}{dx}(\ln x^2)\,x^2$

$= 2x\ln x^2 + \frac{2x}{x^2}\,(x^2)$

$= 2x\ln x^2 + 2x \quad (= 4x\ln x + 2x$

58. $f(x) = \log(\sqrt[3]{\tan x^2}\ )$

$= \frac{1}{\ln 10}\left(\frac{d}{dx}\sqrt[3]{\tan x^2}\right)\left(1/\sqrt[3]{\tan x^2}\right)$

$= \dfrac{\frac{1}{3}(\tan x^2)^{-2/3}(\sec^2 x^2)(2x)}{(\ln 10)(\sqrt[3]{\tan x^2})} = \dfrac{2x\sec^2 x^2}{3(\ln 10)(\tan x^2)}$

GRADE

62. $\frac{d}{dx}\ln[f(x)/x] = \frac{d}{dx}\left[\frac{f(x)}{x}\right]\ \left(1/(f(x)/x)\right)$

$= (\frac{xf'(x) - f(x)}{x^2})(\frac{x}{f(x)}) = \frac{xf'(x) - f(x)}{xf(x)} = \frac{f'(x)}{f(x)} - \frac{1}{x}.$

or use $\frac{d}{dx}[\ln f(x) - \ln x] = \frac{f'(x)}{f(x)} - \frac{1}{x}$

64. $f(x) = x^{2x}$

$y = x^{2x}$

$\ln y = \ln x^{2x}$

$\ln y = 2x\ln x$

$\frac{d}{dx}\ln y = \frac{d}{dx}\,2x\ln x$

$\frac{1}{y}\,y' = 2\ln x + 2x\,(1/x)$

$\frac{1}{y}\,y' = 2\ln x + 2$

$y' = (2\ln x + 2)(y)$

$y' = (2\ln x + 2)(x^{2x})$

GRADE

66. $f(x) = (\ln x)^{3x}$

$$y = (\ln x)^{3x}$$

$$\ln y = \ln(\ln x)^{3x}$$

$$\ln y = 3x\ln(\ln x)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(3x\ln(\ln x))$$

$$\frac{1}{y}y' = 3\ln(\ln x) + 3x\left(\frac{d/dx(\ln x)}{\ln x}\right)$$

$$\frac{1}{y}y' = 3\ln(\ln x) + 3x\left(\frac{1}{x\ln x}\right)$$

$$y' = \left(3\ln(\ln x) + 3/\ln x\right)y$$

$$y' = \left(3\ln(\ln x) + 3/\ln x\right)(\ln x)^{3x}$$

76. $y = \frac{e^{x-1}\sin^2 x}{(x^2+5)^{2x}}$

$$\ln y = \ln\left(\frac{e^{x-1}\sin^2 x}{(x^2+5)^{2x}}\right)$$

$$\ln y = \ln(e^{x-1}\sin^2 x) - \ln(x^2+5)^{2x}$$

$$\ln y = \ln e^{x-1} + \ln(\sin^2 x) - 2x\ln(x^2+5)$$

$$\ln y = x - 1 + 2\ln\sin x - 2x\ln(x^2+5)$$

$$1/y\ y' = 1 + 2(\cos x/\sin x) - \left(2\ln(x^2+5) + 2x\left(\frac{2x}{x^2+5}\right)\right)$$

$$1/y\ y' = 1 + 2\cot x - 2\ln(x^2+5) - 4x^2/x^2+5$$

$$y' = (1 + 2\cot x - 2\ln(x^2+5) - 4x^2/x^2+5)\,y$$

$$y' = (1 + 2\cot x - 2\ln(x^2+5) - 4x^2/x^2+5)\left(e^{x-1}\sin^2 x/(x^2+5)^{2x}\right)$$