Homework #6 *Answers need not be reduced* Due W10 10/19

Section 4.3

6. $f(x) = 2(3x^2-2x^3)(1-5x^2)$

$f'(x) = 2[(6x-6x^2)(1-5x^2) + (-10x)(3x^2-2x^3)] = 2[6x-30x^3-6x^2+30x^4-30x^3+20x^4]$

$= 2(50x^4-60x^3-6x^2+6x) = 100x^4-120x^3-12x^2-12x$

18. $f(x) = (1-2x)(1+2x)$, $x=2$

$f'(x) = (-2)(1+2x) + (2)(1-2x)$

$= -2-4x+2-4x = -8x$

$f'(2) = -8(2) = -16$

$f(2) = (1-2(2))(1+2(2))$

$= (1-4)(1+4)$

$= (-3)(5) = -15$

$y-y_1 = m(x-x_1)$

$y-(-15) = -16(x-2)$

$y+15 = -16x+32$

$y = -16x+17$

36. $f(2)=-4$, $g(2)=3$, $f'(2)=1$, $g'(2)=-2$

$(f^2+g^2)'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$(f^2+g^2)'(2) = 2f(2)f'(2) + 2g(2)g'(2)$

$= 2(-4)(1) + 2(3)(-2)$

$= -8-12 = -20$

44. $y = [-2f(x)-3g(x)]g(x) + \frac{2g(x)}{3}$

$y' = (-2f'(x)-3g'(x))g(x) + (-2f(x)-3g(x))g'(x) + \frac{2}{3}g'(x) = -2f'(x)g(x) - 6g'(x)g(x) - 2f(x)g'(x) + \frac{2}{3}g'(x)$

52. $f(x) = \frac{3x^3+2x-1}{5x^2-2x+1}$

$f'(x) = \frac{(9x^2+2)(5x^2-2x+1) - (10x-2)(3x^3+2x-1)}{(5x^2-2x+1)^2}$

78. $f(x) = \frac{ax^2}{k^2+x^2}$

$f'(x) = \frac{2ax(k^2+x^2) - 2x(ax^2)}{(k^2+x^2)^2} = \frac{2axk^2+2ax^3-2ax^3}{(k^2+x^2)^2} = \frac{2axk^2}{(k^2+x^2)^2}$

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90. $y = \frac{x^2}{f(x)-g(x)}$

$y' = \frac{2x(f(x)-g(x)) - x^2(f'(x)-g'(x))}{(f(x)-g(x))^2}$

Section 4.4

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4. $f(x) = (2x^2 - 3x)^2$

$f'(x) = 2(2x^2-3x)(4x-3)$

14. $f(x) = \sqrt{x^2-1} \,/\, (1+\sqrt{x^2+1})$

$$f'(x) = \frac{\frac{1}{2}(x^2-1)^{-1/2}(2x)(1+\sqrt{x^2+1}) - (\frac{1}{2}(x^2+1)^{-1/2}(2x)(\sqrt{x^2-1})}{(1+\sqrt{x^2+1})^2}$$

$$f'(x) = \frac{x(x^2-1)^{-1/2}(1+\sqrt{x^2+1}) - x(x^2+1)^{-1/2}(\sqrt{x^2-1})}{(1+\sqrt{x^2+1})^2}$$

34. $\frac{d}{dx} f(x) = 2x+1$

(a) $\frac{d}{dx} f(x^2) = f'(x^2)(2x)$

$= (2x^2+1)(2x)$

$\frac{d}{dx} f((-1)^2) = (2(-1)^2+1)(2(-1))$

$= (2+1)(-2)$

$= (3)(-2) = -6$

(b) $\frac{d}{dx} f(\sqrt{x}) = f'(\sqrt{x})(\frac{1}{2}x^{-1/2})$

$= (2(\sqrt{x})+1)(\frac{1}{2}x^{-1/2})$

$\frac{d}{dx}(f(\sqrt{4})) = (2(\sqrt{4})+1)(\frac{1}{2}(4)^{-1/2})$

$= (2(2)+1)(1/2(1/2))$

$= (4+1)(1/4) = 5/4$

52. $\frac{1}{xy} - y^2 = 2$

$\frac{d}{dx}(\frac{1}{xy} - y^2) = \frac{d}{dx}(2)$

$\frac{d}{dx}(xy)^{-1} - 2y\frac{dy}{dx} = 0$

$-(xy)^{-2}(y + x\frac{dy}{dx}) - 2y\frac{dy}{dx} = 0$

$-\frac{y}{(xy)^2} - \frac{x}{(xy)^2}\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-\frac{x}{x^2y^2} - 2y) = \frac{y}{x^2y^2}$

$dy/dx = \frac{y}{x^2y^2}(-\frac{x}{x^2y^2} - 2y)^{-1}$

$= \frac{y}{x^2y^2}(-\frac{x}{x^2y^2} - \frac{2y^3x^2}{x^2y^2})^{-1}$

$= \frac{y}{-x-2x^2y^3} = -\frac{y}{x(1+2xy^3)}$

58. (a) $y^2 = x^2 - x^4$

$2y\frac{dy}{dx} = 2x - 4x^3$

$\frac{dy}{dx} = \frac{2x-4x^3}{2y} = \frac{x-2x^3}{y}\Big]_{(\frac{1}{2}, \frac{1}{4}\sqrt{3})}$

$\frac{dy}{dx} = \frac{2(1/2) - 4(1/2)^3}{2(1/4\sqrt{3})}$

$= \frac{1-1/4}{1/2\sqrt{3}} = \frac{3/4}{1/2\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{2(3)} = \frac{\sqrt{3}}{2}.$

(b) Graph.

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64. $u^2 + v^3 = 12 \qquad dv/dt = 2,\ v = 2,\ u > 0$

$2u\frac{du}{dt} + 3v^2\frac{dv}{dt} = 0$

$2(2)(\frac{du}{dt}) + 3(2)^2(2) = 0$

$4(\frac{du}{dt}) + 24 = 0$

$4\frac{du}{dt} = -24$

$\frac{du}{dt} = -6$

$u^2 + (2)^3 = 12$

$u^2 + 8 = 12$

$u^2 = 4$

$u = \pm 2 \Rightarrow u = 2$, since $u > 0$.

70. $V = \frac{1}{3}\pi r^2 h \qquad \frac{dV}{dt} = 5\ ft^3/min$

$h = 6ft,\ r = 3ft \Rightarrow r = \frac{1}{2}h$, by Similar Triangles

$\frac{dh}{dt} = ?$ when $h = 2ft$

$V = \frac{1}{3}\pi(\frac{1}{2}h)^2 h$

$V = \frac{1}{3}\pi(\frac{1}{4}h^2)(h)$

$V = \frac{1}{3}\pi(\frac{1}{4}h^3)$

$\frac{dV}{dt} = \pi(\frac{1}{4}h^2)\frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$5 = \frac{1}{4}\pi(2)^2\frac{dh}{dt}$

$5 = \pi\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 5/\pi$

80. $f(x) = \frac{x}{x+1}$

$f'(x) = \frac{1(x+1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2} = (x+1)^{-2}$

$f''(x) = -2(x+1)^{-3}(1) = -\frac{2}{(x+1)^3}$

85. $p(x) = ax^2 + bx + c \qquad p'(x) = 2ax + b \qquad p''(x) = 2a$

$3 = a(0)^2 + b(0) + c \qquad 2 = 2a(0) + b \qquad 6 = 2a$

$\underline{3 = c} \qquad \underline{2 = b} \qquad \underline{3 = a}$

So, $p(x) = 3x^2 + 2x + 3$

86. (a) $s(t) = t^2 - 3t$

$v(t) = 2t - 3 \Rightarrow v(1) = 2(1) - 3 = -1$

$a(t) = 2 \Rightarrow a(1) = 2$.

(b) $s(t) = \sqrt{t^2+1}$

$v(t) = \frac{1}{2}(t^2+1)^{-1/2}(2t) \Rightarrow v(1) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$a(t) = -\frac{t}{2}(t^2+1)^{-3/2} + (t^2+1)^{-1/2}$

$\Rightarrow a(1) = -\frac{1}{2}(2)^{-3/2} + (2)^{-1/2}$

(c) $s(t) = t^4 - 2t$

$v(t) = 4t^3 - 2 \Rightarrow v(1) = 4 - 2 = 2$

$a(t) = 12t^2 \Rightarrow a(1) = 12$.

ection 4.5

6. $f(x) = \sec x - \csc x$

$f'(x) = \sec x \tan x - (-\csc x \cot x)$

$= \sec x \tan x + \csc x \cot x$

18. $f(x) = \cos^2(x^2 - 1)$

$f'(x) = 2\cos(x^2-1)(-\sin(x^2-1)(2x))$

$= -4x\cos(x^2-1)\sin(x^2-1)$

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26. $f(x) = -\tan(3x^3 - 4x)$

$f'(x) = -\sec^2(3x^3-4x)(9x^2-4) = -(9x^2-4)\sec^2(3x^3-4x).$

34. $f(x) = \cot(2x) / \tan(4x)$

$$f'(x) = \frac{-\csc^2(2x)(2)\tan(4x) - \sec^2(4x)(4)\cot(2x)}{(\tan(4x))^2}$$

$$= \frac{-2\csc^2(2x)\tan(4x) - 4\sec^2(4x)\cot(2x)}{\tan^2(4x)}$$

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40 $f(x) = \sec x \cos x = \frac{1}{\cos x}\cos x = 1$

$f'(x) = \sec x \tan x \cos x + \sec x(-\sin x)$

$= \sec x \sin x - \sec x \sin x = 0.$

60. $y = \cos^2 x$

$y' = 2\cos x(-\sin x) = -2\cos x \sin x$

$0 = -2\cos x \sin x$

$0 = \cos x \sin x$

$0 = \cos x \qquad 0 = \sin x$

$x = \frac{(2k+1)\pi}{2} \qquad x = k\pi,\ k \in \mathbb{Z}$

Thus, y has a horizontal tangent at $\left(\frac{(2k+1)\pi}{2}, 0\right), (k\pi, 1)$ where $k \in \mathbb{Z}$.

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62. $\frac{d}{dx}\cot x = \frac{d}{dx}\frac{\cos x}{\sin x} = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x.$$

↳ MUST use quotient rule.