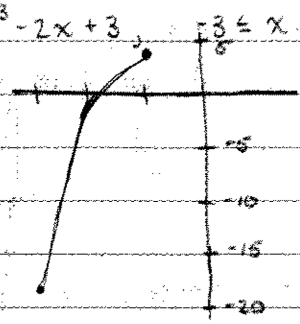
Homework #5 Due W/D 10/12

Section 3.5

2. $f(x) = x^3 - 2x + 3$, $-3 \le x \le -1$

(a)



(b) Since $f(-3) < 0 < f(-1)$, by the IVT, there exists $-3 < c < -1$ s.t. $f(c) = 0$.

* Continuous because it is a polynomial

6. $f(x) = \cos x - x$

$f(0) = \cos(0) - 0 = 1 - 0 = 1 > 0$

$f(1) = \cos(1) - 1 < 1 - 1 = 0$.

~~continuous because trig function - poly.~~

Therefore, since $f(1) < 0 < f(0)$, by the IVT, there exists $0 < c < 1$ s.t. $f(c) = 0$

GRADE

14. Since $f(x)$ is a polynomial, it is continuous $\forall x \in \mathbb{R}$.

So, since $f(x)$ is of odd degree, we consider two cases.

First, where the leading coefficient is positive. Then,

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

So, we can choose a, b large enough so that $f(a) < 0 < f(b)$. Then, by the IVT, there exists $a < c < b$, so that $f(c) = 0$.

Now, we consider when the leading coefficient is negative. Then,

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$.

So again, there exists a c s.t. $f(c) = 0$.

Section 4.1

14. (a) $f(x) = -2x^2$, $x = 1$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{-2(1+h)^2 + 2(1)^2}{h} = \lim_{h \to 0} \frac{-2(1+2h+h^2) + 2}{h}$$

$$= \lim_{h \to 0} \frac{-2 - 4h - 2h^2 + 2}{h} = \lim_{h \to 0} \frac{h(-4-2h)}{h} = \lim_{h \to 0} (-4-2h) = -4.$$

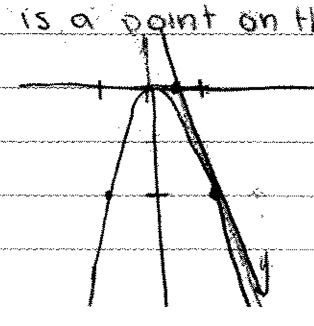
(b) $f(1) = -2(1)^2 = -2(1) = -2$, so $(1, -2)$ is a point on the graph

$y - (-2) = -4(x - 1)$

$y + 2 = -4x + 4$

$\underline{y = -4x + 2}$ → Tangent Line

(c)



16. (a) $f(x) = 1/x$, $x = 2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{(2-2-h)/(2(2+h))}{h}$$

$$= \lim_{h\to 0} \frac{-h}{h(4+2h)} = \lim_{h\to 0} \frac{-1}{4+2h} = -1/4$$

(b) $f(2) = 1/2$, so $(2, 1/2)$ is a point on the graph of $f(x)$.

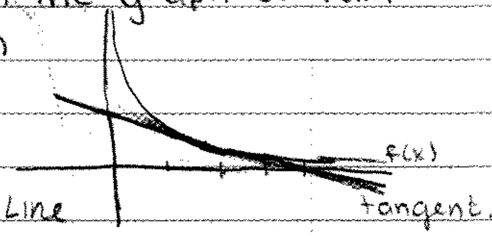
Normal Line has slope 4

$y - 1/2 = 4(x-2)$

$y - 1/2 = 4x - 8$

$\underline{y = 4x - 15/2}$

(c)



Graph Tangent Line

22. $f(x) = x^2 - 3x + 1$ $(2, -1)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 1 - x^2 + 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} = \lim_{x\to 0} \frac{h(2x+h-3)}{h}$$

$$= \lim_{h\to 0} 2x + h - 3 = 2x - 3.$$

$f'(2) = 2(2) - 3 = 4 - 3 = 1.$

$f(2) = (2)^2 - 3(2) + 1 = 4 - 6 + 1 = -1$

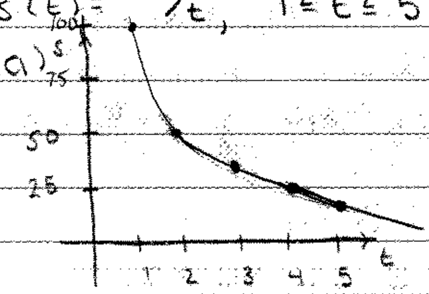
$y - (-1) = 1(x-2)$

$y + 1 = x - 2$

$\underline{y = x - 3}$

32. $s(t) = 100/t$, $1 \le t \le 5$

(a)



(b) $AV = \frac{f(5) - f(1)}{5-1} = \frac{20 - 100}{4} = \frac{-80}{4} = -20.$

Secant Line through $f(1)$, $f(5)$

(c) $IV = s'(2) = \lim_{h\to 0} \frac{100/(2+h) - 100/2}{h} = \lim_{h\to 0} \frac{200 - 200 - 100h}{h(2)(2+h)}$

$= \lim_{h\to 0} -\frac{100h}{2h(2+h)} = \lim_{h\to 0} -\frac{50}{2+h} = -\frac{50}{2} = -25$

Slope at $t = 2$. (of tangent line)

The speed at $t=0$ is 25 km/hr

36. $dN/dt = rN$

(a) Per Capita Growth Rate : $\frac{dN}{dt} \cdot \frac{1}{N} = r$

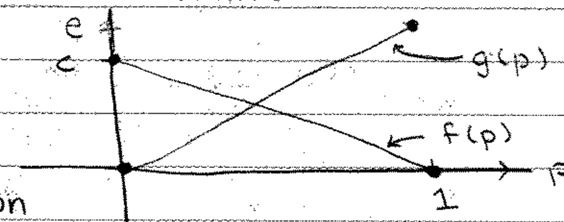
(b) Greater; $r > 0 \Rightarrow$ Growth rate $> 0 \Rightarrow$ Population Grows

$\therefore N(1) > 20$

38. $dp/dt = c(1-p) - ep$, $c, e > 0$ constant

(a) $f(p) = c(1-p)$

$g(p) = ep$



(or Something similar)

If e, c are positive, we have an intersection point because

$0 = f(1) < f(0) = c$, and

$0 = g(0) < g(1) = e$ imply that,

$-c = (g-f)(0) < 0 < (g-f)(1) = e$

So, by the IVT, there exists $t \in (0,1)$ s.t.

$(g-f)(t) = 0$, which implies that $g(t) = f(t)$.

To find intesection point, solve $\underbrace{c(1-p) = ep}_{dp/dt = 0}$ for p.

So, $c - cp = ep$

$c = ep + cp$

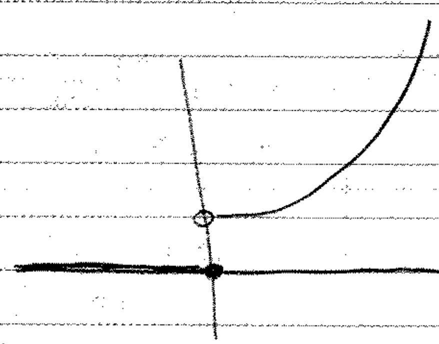
$c = (e+c)p \Rightarrow p = c/e+c$.

This is where the gain equals the loss, no population change.

(b) As c increases, the intersection point becomes closer and closer to $p=1$. (can see by $\lim_{c \to \infty} \frac{c}{e+c} = 1$)

In otherwords, the populations are being replenished just as quickly as they are being lost by extinction. The extinction rate becomes the dominant term

EP1. $f(x) = \begin{cases} 0 & x \le 0 \\ x^2+1 & x > 0 \end{cases}$



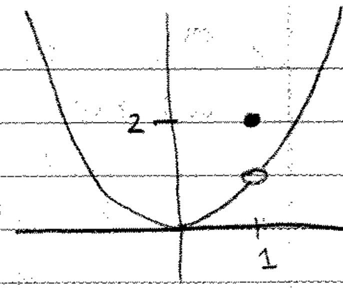
$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2+1-0}{h}$

So, $\lim_{h \to 0^+} \frac{h^2+1}{h}$ DNE.

$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{0-0}{h} = 0$

Since the limits are not equal, $f'(0)$ DNE.

EP2. $g(x) = \begin{cases} \frac{x^2(x-1)}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases} = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$



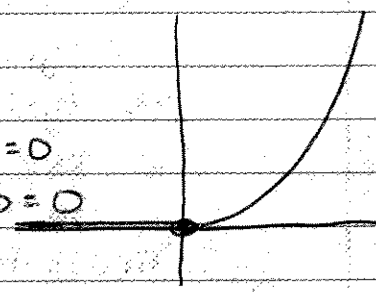
$\lim_{h \to 0^+} \frac{g(1+h) - g(1)}{h} = \lim_{h \to 0^+} \frac{(1+h)^2 - 2}{h}$

$= \lim_{h \to 0^+} \frac{1 + 2h + h^2 - 2}{h} = \lim_{h \to 0^+} \frac{h^2 + 2h - 1}{h}$ DNE

Similarly, $\lim_{h \to 0^-} \frac{g(1+h) - g(1)}{h}$ DNE.

Therefore, $g'(1)$ DNE.

GRADE EP3. $f(x) = \begin{cases} 0 & x \leq 0 \\ x^2 & x > 0 \end{cases}$



$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 - 0}{h} = \lim_{h \to 0^+} h = 0$

$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{0 - 0}{h} = \lim_{h \to 0^-} 0 = 0$

So, since $\lim_{h \to 0^-} = \lim_{h \to 0^+}$, $f'(0)$ exists.

Section 4.2 2. $f(x) = 3x^2 - 4x^4 \Rightarrow f'(x) = 6x - 16x^3$

12. $f(x) = 2x^3 \cos\frac{\pi}{4} + \cos\frac{\pi}{4}$

$f'(x) = 6x^2 \cos\pi/4 = 6x^2 (\sqrt{2}/2) = 6x^2/\sqrt{2}$

30. $f(x) = \frac{r+x}{rs^2} - rsx + (r+s)x - rs$, $r, s \neq 0$ constant

$= \frac{r}{rs^2} + \frac{x}{rs^2} - rsx + (r+s)x - rs$

$f'(x) = \frac{1}{rs^2} - rs + (r+s)$

32. $f(N) = \frac{bN^2 + N}{k+b}$, $k, b > 0$ constant

$f'(N) = \frac{1}{k+b}(2bN + 1)$

38. $g(N) = rN(1 - N/k)$ $r, K > 0$ constant

$= rN - \frac{r}{k}N^2$

$g'(N) = r - 2(r/k)N$

GRADE 40. $g(N) = rN(a - N)(1 - N/k)$ $r, a, K > 0$ constant.

$= (rNa - rN^2)(1 - N/k)$

$= rNa - rN^2 - arN^2/k + rN^3/k$

$g'(N) = ra - 2rN - 2\left(\frac{ar}{k}\right)N + 3\left(\frac{r}{k}\right)N^2$

GRADE

44. $y = -2x^3 - 3x + 1$, at $x = 1$

$y = -2(1)^3 - 3(1) + 1 = -2 - 3 + 1 = -4$.

$\Rightarrow (1, -4)$ is a point on the graph.

$y' = -6x^2 - 3 \Rightarrow m = -6(1)^2 - 3 = -6 - 3 = -9$

So, $y - (-4) = -9(x - 1)$

$y + 4 = -9x + 9$

$y = -9x + 5$ ⟵ Tangent Line.

In standard form: $\underline{9x + y - 5 = 0}$ ← Must be in this form.

58. $f(x) = x^2/a+1$, $x = a > 0$, constant.

$f(a) = a^2/a+1 \Rightarrow (a, a^2/a+1)$ is a point on the graph

$f'(x) = 2x/a+1 \Rightarrow m = 2a/a+1$ is the slope

So, $y - a^2/a+1 = (2a/a+1)(x - a)$

$y - a^2/a+1 = (2a/a+1)x - 2a^2/a+1$

$y = \frac{2a}{a+1}x - \frac{a^2}{a+1}$ ⟵ Tangent Line.

GRADE

68. $f(x) = -4x^4 + x^3$

$f'(x) = -16x^3 + 3x^2$

$0 = -16x^3 + 3x^2$

$0 = x^2(-16x + 3)$

$x = 0$, $x = 3/16$

So, $f(0) = 0$

$f(3/16) = -4(3/16)^4 + (3/16)^3$

$= -4(81/65536) + 27/4096$

$= -81/16384 + 108/16384$

$= 27/16384$

Thus, $(0,0)$ and $(3/16, 27/16384)$ ← Must have coordinates

76. $y - 2x = 1$

$y = 2x + 1$, so $m = 2$.

$f(x) = 2x^3 - 4x + 1$

$f'(x) = 6x^2 - 4$

want $2 = 6x^2 - 4$

$1 = 3x^2 - 2$

$3 = 3x^2$

$1 = x^2$

$x = \pm 1$

Points are:

$(1, -1)$ and $(-1, 3)$

Yes, there is more than one.