Homework #4 Due w/o 10/5

Section 3.2

6. $\lim_{x\to -2}\left(\frac{2x^2+x-6}{x+2}\right) = \lim_{x\to -2}\left(\frac{(2x-3)(x+2)}{x+2}\right) = \lim_{x\to -2}(2x-3) = 2(-2)-3 = -7.$

Thus, ① $f(x)$ is defined at $x=-2$ (see piecewise function)

② $\lim_{x\to -2} f(x) = -7$ (and thus exists)

③ $\lim_{x\to -2} f(x) = -7 = f(-2)$.

Therefore, $f(x)$ is continuous at $x=-2$.

8. must find a s.t. $f(1) = a = \lim_{x\to 1} f(x) = \lim_{x\to 1}\left(\frac{x^2+x-2}{x-1}\right)$

So, $\lim_{x\to 1}\left(\frac{x^2+x-2}{x-1}\right) = \lim_{x\to 1}\left(\frac{(x-1)(x+2)}{x-1}\right) = \lim_{x\to 1}(x+2) = (1)+2 = 3$.

Thus, let $\underline{a=3}$.

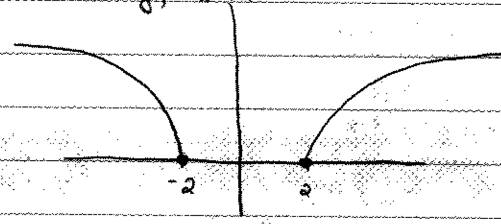
GRADE

16. (a) Note that $f(x)=\sqrt{x^2-4}$ is defined near $x=2$ on the right.

And $\lim_{x\to 2^+} f(x) = \sqrt{0} = 0 = f(2)$

Similarly, $\lim_{x\to -2^-} f(x) = \sqrt{0} = 0 = f(-2)$.

(b)

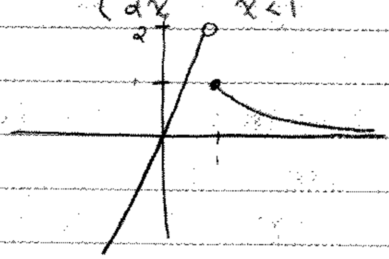


(c) No, because the function is not defined to the left of $x=2$ or to the right of $x=-2$.

22. $f(x) = \ln(x-2)$ Continuous for $x\in(2,\infty)$

23. $f(x) = \ln\left(\frac{x}{x+1}\right)$ Continuous for $x\in(-\infty,-1)\cup(0,\infty)$

28. (a) $f(x) = \begin{cases} 1/x & x\geq 1 \\ 2x & x<1 \end{cases}$



(b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(2x+c) = 1$

$\Rightarrow 2(1)+c = 1 \Rightarrow c=-1.$

32. $\lim_{x\to -\pi/2} \frac{1+\tan^2 x}{\sec^2 x} = \lim_{x\to -\pi/2}\frac{\sec^2 x}{\sec^2 x} = \lim_{x\to -\pi/2} 1 = 1$

36. $\sqrt{x^3+4x-1}$ is continuous at $x=1$, so

$\lim_{x\to 1}\sqrt{x^3+4x-1} = \sqrt{(1)^3+4(1)-1} = \sqrt{1+4-1} = \sqrt{4} = 2$

GRADE

42. Since e^x, e^{-x}, constant functions are all continuous at $x=0$, $\frac{e^{-x}-e^x}{e^{-x}+1}$ is continuous at 0 (since $\lim_{x\to0}(e^{-x}+1)\neq 0$).

Thus, $\lim_{x\to0}\frac{e^{-x}-e^x}{e^{-x}+1} = \frac{e^{-(0)}-e^{(0)}}{e^{-(0)}+1} = \frac{1-1}{1+1} = 0/2 = 0$.

46. $\lim_{x\to0}\frac{5-\sqrt{25+x^2}}{2x^2} = \lim_{x\to0}\left(\left(\frac{5-\sqrt{25+x^2}}{2x^2}\right)\left(\frac{5+\sqrt{25+x^2}}{5+\sqrt{25+x^2}}\right)\right)$

$= \lim_{x\to0}\left(\frac{25-(25+x^2)}{10x^2+2x^2\sqrt{25+x^2}}\right) = \lim_{x\to0}\frac{\cancel{x^2}}{\cancel{x^2}(10+2\sqrt{25+x^2})}$

$= \lim_{x\to0}\left(1/10+2\sqrt{25+x^2}\right) = 1/10+2\sqrt{25+0} = 1/10+2\sqrt{25} = 1/10+2(5)$

$= 1/10+10 = 1/20$

Section 3.3

2. $\lim_{x\to\infty}\left(\frac{x^2+3}{5x^2-2x+1}\right) = \lim_{x\to\infty}\left(\frac{x^2/x^2+3/x^2}{5x^2/x^2-2x/x^2+1/x^2}\right) = \lim_{x\to\infty}\left(\frac{1+3/x^2}{5-2/x+1/x^2}\right) = 1/5$

6. $\lim_{x\to\infty}\left(\frac{5x^3-1}{4x^4+1}\right) = \lim_{x\to\infty}\left(\frac{5x^3/x^4-1/x^4}{4x^4/x^4+1/x^4}\right) = \lim_{x\to\infty}\left(\frac{5/x-1/x^4}{4+1/x^4}\right) = \frac{0-0}{4+0} = 0/4 = 0$

12. Using Division,

$$\begin{array}{r} x^2+2x-\frac{2x+5}{x^2-x+2} \\ x^2-x+2\ \overline{)\ x^4+2x-5} \\ -x^4+2x^3\oplus 2x^2 \\ \hline 2x^3-2x^2+2x \\ -2x^3+2x^2\oplus 4x \\ \hline -2x-5 \end{array}$$

So, $\lim_{x\to-\infty}\left(\frac{x^4+2x-5}{x^2-x+2}\right) = \lim_{x\to-\infty}\left(x^2+2x-\frac{2x+5}{x^2-x+2}\right)$ DNE.

GRADE

16. $\lim_{x\to\infty}\frac{e^x}{2-e^x} = \lim_{x\to\infty}\left(\frac{e^x/e^x}{2/e^x-e^x/e^x}\right) = \lim_{x\to\infty}\left(\frac{1}{2/e^x-1}\right) = \frac{1}{0-1} = \frac{1}{-1} = -1$

25. $\lim_{N\to\infty}r(N) = \lim_{N\to\infty}\left(a\frac{N}{k+N}\right) = \lim_{N\to\infty}\left(\frac{aN/N}{k/N+N/N}\right) = \lim_{N\to\infty}\left(\frac{a}{k/N+1}\right) = \frac{a}{0+1} = a$

Section 3.4

2. $f(x) = x^2\sin 1/x$, $x\neq 0$

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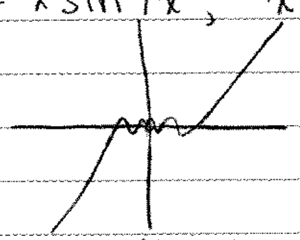
(a)



(b) Note that $-1\le\sin(1/x)\le 1$

Then, since $x^2\geq 0$, we have
$-x^2\le x^2\sin(1/x)\le x^2$.

(c) $\lim_{x\to0}(-x^2) = \lim_{x\to0}(x^2) = 0$.

Thus, by the Squeeze Theorem, $\lim_{x\to0}x^2\sin(1/x) = 0$.

GRADE

6. $\lim_{x\to 0} \frac{\sin(2x)}{3x} = \lim_{x\to 0} \left(\frac{2\sin(2x)}{3(2x)}\right) = \lim_{x\to 0} \left(\frac{2}{3}\left(\frac{\sin 2x}{2x}\right)\right) = \frac{2}{3} \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right) = \frac{2}{3}(1) = \frac{2}{3}$.

10. $\lim_{x\to 0} \frac{\sin(-\pi x/2)}{2x} = \lim_{x\to 0} \left(\frac{-\pi/2}{2}\left(\frac{\sin(-\pi x/2)}{-\pi x/2}\right)\right) = -\frac{\pi}{4}\left(\lim_{x\to 0} \frac{\sin(-\pi x/2)}{-\pi x/2}\right) = -\frac{\pi}{4}(1) = -\frac{\pi}{4}$.

14. $\lim_{x\to 0} \frac{1-\cos^2 x}{x^2} = \lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 = 1^2 = 1$

16. $\lim_{x\to 0} \frac{1-\cos(2x)}{3x} = \lim_{x\to 0} \left(\frac{2(1-\cos 2x)}{3(2x)}\right) = \frac{2}{3}\left(\lim_{x\to 0}\left(\frac{1-\cos 2x}{2x}\right)\right) = \frac{2}{3}(0) = 0$.