Homework #2 Due w/o 9/21

Section 2.1

16. $t=0 \Rightarrow P=1$
$t=42 \Rightarrow P=2$
$t=84 \Rightarrow P=4$

Let $t=42x$, where x is measured in units of time

So, $P(t)=2^x=2^{t/42}$. Want $P=512$

$512=2^{t/42}$

$\log_2 512=\log_2(2^{t/42})$

$\log_2 512=t/42$

$9=t/42 \Rightarrow t=378$ minutes

22. $P(t)=72 \cdot 3^t$

36.

N_0	N_1	N_2	N_3	N_4	N_5
5	10	20	40	80	160

$N_{t+1}=2N_t$
$N_0=5$

42.

N_0	N_1	N_2	N_3	N_4	N_5
4096	2048	1024	512	256	128

$N_{t+1}=\frac{1}{2}N_t$
$N_0=4096$

52. $N_{t+1}=5N_t$, $N_0=17$

$N_1=5\cdot17$, $N_2=5^2\cdot17 \Rightarrow N_t=5^t\cdot17$

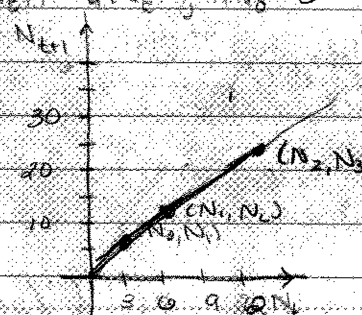
GRADE

56. $N_{t+1}=\frac{1}{3}N_t$, $N_0=3500$

$N_1=\frac{1}{3}(3500)$, $N_2=(\frac{1}{3})^2(3500) \Rightarrow N_t=(\frac{1}{3})^t(3500)$

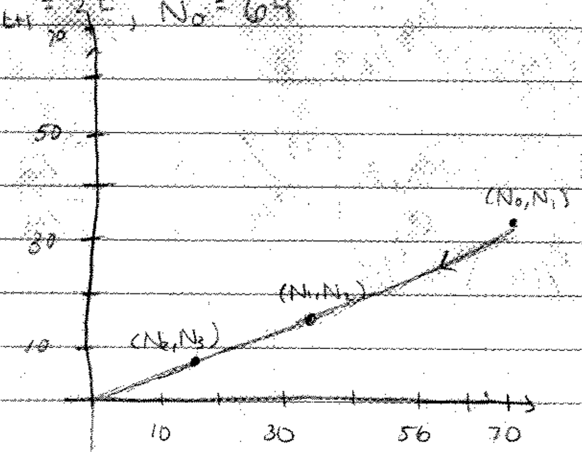
60. $N_{t+1}=2N_t$; $N_0=3$

(N_t, N_{t+1}): $t=0 \Rightarrow (3,6)$
$t=1 \Rightarrow (6,12)$
$t=2 \Rightarrow (12,24)$



64. $N_{t+1}=\frac{1}{2}t$; $N_0=64$

(N_t, N_{t+1}): $t=0 \Rightarrow (64,32)$
$t=1 \Rightarrow (32,16)$
$t=2 \Rightarrow (16,8)$



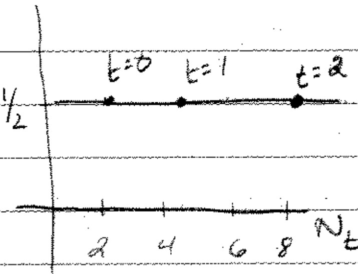
68. $N_t/N_{t+1} = 1/2$, $N_0 = 2$.

$N_{t+1} = 2N_t$

$(N_t, N_t/N_{t+1})$: $t=0 \Rightarrow (2, 1/2)$

$t=1 \Rightarrow (4, 1/2)$

$t=2 \Rightarrow (8, 1/2)$



75. (a) No, no room for growth.

(b) Yes, has room to grow exponentially — but nesting sites may be ruined

(c) Yes, has room to grow exponentially, assuming not 100%.

76. Many reasons: Example: the people colonizing the area provided a route to pollination to a larger area. Thus the pines spread exponentially with that movement until they filled the limiting area, and utilized all the resources.

ection 2.2

16. $a_0 = (-1)^0/(0+1)^2 = 1/1^2 = 1$ $\quad a_1 = (-1)^1/(1+1)^2 = -1/2^2 = -1/4$

$a_2 = (-1)^2/(2+1)^2 = 1/3^2 = 1/9$ $\quad a_3 = (-1)^3/(3+1)^2 = -1/4^2 = -1/16$

$a_4 = (-1)^4/(4+1)^2 = 1/5^2 = 1/25$ $\quad a_5 = (-1)^5/(5+1)^2 = -1/6^2 = -1/36$

16. $\sin \pi/2, -\sin \pi/4, \sin \pi/6, -\sin \pi/8, \sin \pi/10, \ldots$

$-\sin \pi/12, \sin \pi/14, -\sin \pi/16, \sin \pi/18$

22. $1/3, 2/5, 3/7, 4/9, 5/11, \ldots$ $\quad a_n = n+1/(2n+3)$

26. $1/2, -1/8, 1/18, -1/32, 1/50, \ldots$

0 1 2 3 4

+6 +10 +14 +18

$2(1)^2 \quad 2(2)^2 \quad 2(3)^2 \quad 2(4)^2 \quad 2(5)^2$

$\therefore a_n = (-1)^n (1/2(n+1)^2)$

GRADE

32. $a_0 = 2(0)/0+2 = 0/2 = \underline{0}$ $\quad a_1 = 2(1)/(1+2) = \underline{2/3}$

$a_2 = 2(2)/(2+2) = 4/4 = \underline{1}$ $\quad a_3 = 2(3)/(3+2) = \underline{6/5}$

$a_4 = 2(4)/(4+2) = 8/6 = \underline{4/3}$

$\lim_{n\to\infty} a_n = 2$

35. $a_0 = (-1)^0/(0+1) = 1/1 = 1$ $\quad a_1 = (-1)^1/(1+1) = -1/2 = -1/2$

$a_2 = (-1)^2/(2+1) = 1/3$ $\quad a_3 = (-1)^3/(3+1) = -1/4 = -1/4$

$a_4 = (-1)^4/(4+1) = 1/5$

$\lim_{n\to\infty} a_n = 0$.

41. $a_n = n^2/n+1$

$a_0 = 0^2/(0+1) = 0/1 = 0$ $\quad$ $a_1 = 1^2/(1+1) = 1/2$

$a_2 = 2^2/(2+1) = 4/3$ $\quad$ $a_3 = 3^2/(3+1) = 9/4$

$a_4 = 4^2/(4+1) = 16/5$

$\lim_{n\to\infty} a_n$ DNE (i.e. $\to \infty$)

GRADE 46. $a_n = (\frac{1}{2})^n$

$a_0 = (\frac{1}{2})^0 = 1$ $\quad$ $a_1 = (\frac{1}{2})^1 = \frac{1}{2}$

$a_2 = (\frac{1}{2})^2 = 1/4$ $\quad$ $a_3 = (\frac{1}{2})^3 = \frac{1}{8}$

$a_4 = (\frac{1}{2})^4 = \frac{1}{16}$

$\lim_{n\to\infty} a_n = 0$

GRADE 52. $a_n = 1/n^2$, $\varepsilon = 0.001$ $\quad$ $\lim_{n\to\infty} a_n = 0 = a$

$$\left.\begin{array}{l} |\frac{1}{n^2} - a| < 0.001 \\ |\frac{1}{n^2} - 0| < 0.001 \\ |\frac{1}{n^2}| < 0.001 \end{array}\right\} \begin{array}{l} \text{So, } \frac{1}{n^2} < 0.001 \text{, since } n \geq 1 \text{ (always positive)} \\ \frac{1}{0.001} < n^2 \\ 1000 < n^2 \end{array}$$

$31.62 \approx \sqrt{1000} < n.$

Choose $N = 31$. Then if $n > N$, (an integer), $|\frac{1}{n^2}| < 0.001$.

56. $a_n = (-1)^n/n$, $\varepsilon = 0.001$ $\quad$ $\lim_{n\to\infty} a_n = 0 = a$

$$\left.\begin{array}{l} |\frac{(-1)^n}{n} - a| < 0.001 \\ |\frac{(-1)^n}{n} - 0| < 0.001 \\ |\frac{(-1)^n}{n}| < 0.001 \end{array}\right\} \begin{array}{l} \text{So, } \frac{1}{n} < 0.001 \\ \frac{1}{.001} < n \\ 1000 < n. \end{array}$$

Choose $N = 1000$. Then if $n > N$, $|\frac{(-1)^n}{n}| < 0.001$

76. $\lim_{n\to\infty} \left(\frac{3n^2-5}{n^2}\right) = \lim_{n\to\infty} \left(\frac{3n^2}{n^2} - \frac{5}{n^2}\right) = \lim_{n\to\infty} \left(3 - \frac{5}{n^2}\right) = \lim_{n\to\infty} (3) - \lim_{n\to\infty} \left(\frac{5}{n^2}\right)$

$= 3 - 5 \lim_{n\to\infty} \left(\frac{1}{n^2}\right) = 3 - 5(0) = 3 - 0 = 3$

78. $\lim_{n\to\infty} \left(\frac{n+2}{n^2-4}\right) = \lim_{n\to\infty} \left(\frac{\cancel{n+2}}{(\cancel{n+2})(n-2)}\right) = \lim_{n\to\infty} \left(\frac{1}{n-2}\right) = 0.$

94. $a_{n+1} = \frac{1}{3} a_n + \frac{4}{3}$

$a = \frac{1}{3} a + \frac{4}{3}$

$\frac{2}{3} a = \frac{4}{3}$

$2a = 4$

$a = 2.$

100. $a_{n+1} = \frac{3}{a_n - 2}$

$a = \frac{3}{a-2}$

$a(a-2) = 3$

$a^2 - 2a = 3$

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$a - 3 = 0 \quad a + 1 = 0$

$a = 3 \quad a = -1$

104. $a_{n+1} = \frac{1}{3}(a_n + \frac{1}{9})$, $a_0 = 1$

$a = \frac{1}{3}(a + \frac{1}{9})$

$a = \frac{1}{3} a + \frac{1}{27}$

$\frac{2}{3} a = \frac{1}{27}$

$18 a = 1$

$a = 1/18$

Use table to see that $a_n \to 0$ as $n \to \infty$.

108. $a_{n+1} = 2a_n(1 - a_n)$, $a_0 = 0$

$a = 2a(1-a)$

$a = 2a - 2a^2$

$2a^2 - a = 0$

$a(2a-1) = 0$

$a = 0 \quad a = 1/2$

$a_0 = 0$, a fixed point $\Rightarrow \lim_{n\to\infty} a_n = 0.$

GRADE #108 (See Right)