Homework # 12 Due W/D 11/30

Section 6.2

42. $\int (4x^3 + 5x^2)\,dx = x^4 + \frac{5}{3}x^3 + C$

46. $\int \frac{x^2+2x}{2\sqrt{x}}\,dx = \int\left(\frac{x^2}{2\sqrt{x}} + \frac{2x}{2\sqrt{x}}\right)dx = \int\left(\frac{1}{2}x^{3/2} + \sqrt{x}\right)dx = \frac{1}{2}\left(\frac{2}{5}x^{5/2}\right) + \frac{2}{3}x^{3/2} + C$
$= \frac{1}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$

60. $\int 2e^{-x/3}\,dx = -6e^{-x/3} + C$

70. $\int \frac{\cos x}{1-\cos^2 x}\,dx = \int \frac{\cos x}{\sin^2 x}dx = \int\left(\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right)dx = \int \cot x \csc x\,dx$
$= -\csc x + C$

90. $\int 2^x\,dx = \left(\frac{1}{\ln 2}\right)(2^x) + C$

94. $\int (x^{-3} + 3^{-x}) = -\frac{1}{2}x^{-2} - \left(\frac{1}{\ln 3}\right)(3^{-x}) + C$

106. $\int_{-\pi/3}^{\pi/3} 2\cos(x/2)\,dx = 4\sin\left(\frac{x}{2}\right)\Big]_{-\pi/3}^{\pi/3} = 4\sin\left(\frac{1}{2}\left(\frac{\pi}{3}\right)\right) - 4\sin\left(\frac{1}{2}\left(-\frac{\pi}{3}\right)\right)$ By FTC (II) since f continuous
$= 4\sin\left(\frac{\pi}{6}\right) - 4\sin\left(-\frac{\pi}{6}\right)$
$= 4(1/2) - 4(-1/2) = 2 + 2 = 4$

118. $\int_{-1}^{1} e^{-|s|}\,ds = \int_{-1}^{0} e^{-|s|}\,ds + \int_0^1 e^{-|s|}\,ds = -\int_0^{-1} e^{-|s|}\,ds + \int_0^1 e^{-|s|}\,ds$
$= -\int_0^{-1} e^{s}\,ds + \int_0^1 e^{-s}\,ds = -e^s\Big]_0^{-1} + (-e^{-s})\Big]_0^1$ By FTC (II)
$= (-e^{-1} - (-e^0)) + (-e^{-1} - (-e^{-(0)}))$ since f continuous
$= -1/e + 1 - 1/e + 1 = 2 - 2/e$

124. $\lim_{h\to 0} \frac{1}{h}\int_0^h e^x\,dx = \lim_{h\to 0} \frac{\int_0^h e^x\,dx}{h}$ has I.F. $0/0$
So, by L'hôpital, $= \lim_{h\to 0} \frac{d/dh \int_0^h e^x\,dx}{d/dh\ h} = \lim_{h\to 0}\frac{e^h}{1}$
by FTC (I) since e^h is continuous. Evaluating:
$\lim_{h\to 0}\frac{1}{h}\int_0^h e^x\,dx = \lim_{h\to 0} e^h = e^0 = 1.$

Section 6.3 4. $y_1 = x^2$, $y_2 = 2 - x^4$

$x^2 = 2 - x^4$

$x^4 + x^2 - 2 = 0$

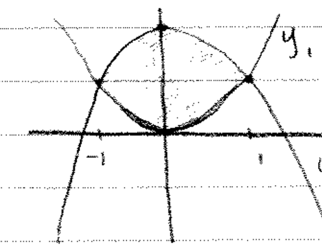
$(x^2+2)(x^2-1) = 0$

$(x^2+2)(x+1)(x-1) = 0$

$\underbrace{x^2+2=0}_{\text{no sol'n}}$ $\quad x+1=0 \quad x-1=0$

$x=-1 \quad x=1$

Find Intersection Points: Graph



$\int_{-1}^{1}(y_2 - y_1)dx = \int_{-1}^{1}(2-x^4-x^2)dx = (2x - \frac{1}{5}x^5 - \frac{1}{3}x^3)\Big]_{-1}^{1}$

$= (2(1) - \frac{1}{5}(1)^5 - \frac{1}{3}(1)^3) - (2(-1) - \frac{1}{5}(-1)^5 - \frac{1}{3}(-1)^3)$

$= (2 - 1/5 - 1/3) - (-2 + 1/5 + 1/3) = 4 - 2/5 - 2/3 = \frac{60-6-10}{15} = \boxed{\frac{44}{15}}$

14. $y = x$

$yx = 1 \Rightarrow x = 1/y$

$y = 1/2$

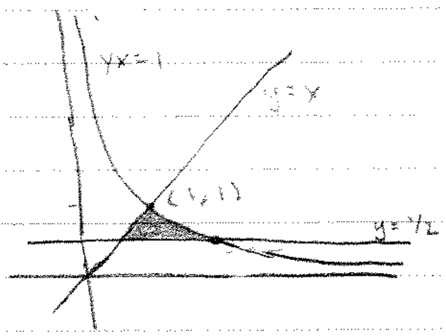
1st Quadrant

Intersection Points:

$y = 1/y \Rightarrow y^2 = 1 \Rightarrow y = 1$

$y = 1/2 \Rightarrow x = 1/(1/2) = 2$

Graph:



$\int_{1/2}^{1}(1/y - y)\,dy = (\ln|y| - \frac{1}{2}y^2)\Big]_{1/2}^{1} = (\ln(1) - \frac{1}{2}(1)^2) - (\ln(1/2) - \frac{1}{2}(\frac{1}{2})^2)$

$= 0 - \frac{1}{2} - \ln(1/2) - 1/8 = \boxed{\ln 2 - 5/8}$

17. $\frac{dN}{dt} = e^{-t}$ $\quad N(0) = 100$

(a) $\int \frac{dN}{dt}dt = \int e^{-t}dt$

$N(t) = -e^{-t} + C$

$100 = -e^{-0} + C$

$100 = -1 + C$

$101 = C$ $\quad \therefore N(t) = -e^{-t} + 101$

(b) Cum. Change $= \int_0^5 e^{-t}dt = -e^{-t}\Big]_0^5 = -e^{-5} + e^{-0} = -1/e^5 + 1 = \frac{e^5 - 1}{e^5}$

(c) Cum. Change $= \int_0^t e^{-t}dt$

Area under the curve of $f(t) = e^{-t}$.

20. $\int_0^t 32\,dt = 32t\Big]_0^t = 32t$

22. $\int_3^5 \frac{dw}{dx}\,dx$ is the cumulative change in weight between ages 3 and 5.

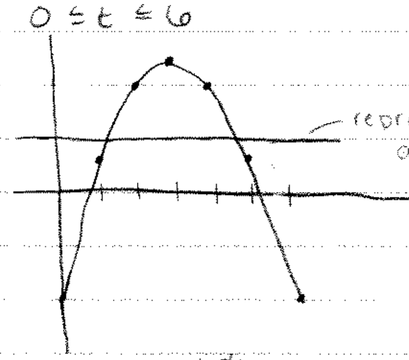
26. $g(t) = e^{-t} \qquad [-1, 1]$

$$g_{avg} = \frac{1}{1-(-1)} \int_{-1}^{1} g(t)\,dt$$
$$= \frac{1}{2} \int_{-1}^{1} e^{-t}\,dt$$
$$= \frac{1}{2}\left[(-e^{-t})\Big]_{-1}^{1}\right]$$
$$= \frac{1}{2}\left[-e^{-1} - (-e^{-(-1)})\right]$$
$$= \frac{1}{2}\left[-1/e + e\right] = \frac{1}{2}\left[\frac{e^2-1}{e}\right] = \frac{e^2-1}{2e}$$

30. The function modeling your speed is continuous. Thus, by the Mean Value Theorem for Definite Integrals, there exists a time $t \in [0, b]$, where b is the total time of your trip, s.t. $f(t)(b-0) = \int_0^b f(x)\,dx$. We also know that the $f(t) = \frac{1}{b-0}\int_0^b f(x)\,dx = f_{avg} = 50$ mph. Therefore, there exists a time s.t. your speed is exactly 50 mph.

32. $v(t) = -(t-3)^2 + 5 \qquad 0 \le t \le 6$

(a) Graph



(b) $$v_{avg} = \frac{1}{6-0}\int_0^6 (-(t-3)^2+5)\,dt = \frac{1}{6}\int_0^6 (-(t^2-6t+9)+5)\,dt$$
$$= \frac{1}{6}\int_0^6 (-t^2+6t-9+5)\,dt = \frac{1}{6}\int_0^6 (-t^2+6t-4)\,dt$$
$$= \frac{1}{6}\left(-\frac{1}{3}t^3+3t^2-4t\right)\Big]_0^6 = \frac{1}{6}\left(-\frac{1}{3}(6)^3+3(6)^2-4(6)\right)$$
$$= -\frac{1}{3}(36)+3(6)-4 = -12+18-4 = 2$$

(c) $-(t-3)^2 + 5 = 2$

$-(t^2-6t+9)+5 = 2$

$-t^2+6t-9+3 = 0$

$t^2-6t+6 = 0$

$$t = \frac{6 \pm \sqrt{36-4(1)(6)}}{2(1)} = \frac{6 \pm \sqrt{36-24}}{2} = \frac{6 \pm \sqrt{12}}{2} = \frac{6 \pm 2\sqrt{3}}{2}$$
$$= 3 \pm \sqrt{3}.$$

There are two points, seen visually as the intersection points of $v(t)$ and the horizontal line $y=2$. At least one point guaranteed by MVT(T) b/c $v(t)$ is cont.