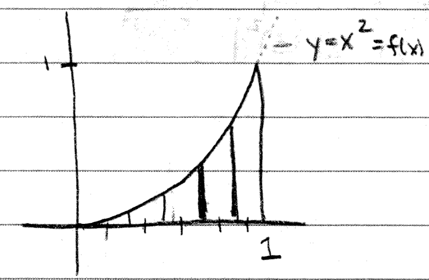
Homework #11 Due on 11/24/20

Section 6.1

2.



5 equal subintervals $\Rightarrow$ width $= 1/5$

Midpoints $\Rightarrow$ Evaluate at middle of interval.

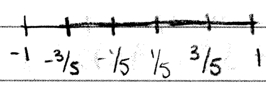
$A \approx (1/5)(f(1/10)) + (1/5)(f(3/10)) + (1/5)(f(5/10))$
$+ (1/5)(f(7/10)) + (1/5)(f(9/10))$
$= \frac{1}{5}(1/100 + 9/100 + 25/100 + 49/100 + 81/100)$
$= \frac{1}{5}(10/100 + 25/100 + 130/100) = 1/5\,(165/100)$
$= 33/100$

$\boxed{\therefore A \approx 33/100}$

10. $\sum_{k=0}^{4} k^x = 0^x + 1^x + 2^x + 3^x + 4^x$

18. $\frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \frac{6}{8} + \frac{7}{9} \boxed{= \sum_{k=1}^{5} \frac{k+2}{k+4} = \sum_{k=3}^{7} \frac{k}{k+2}}$

32.



$f(x) = 1-x^2$

Left Endpoints.

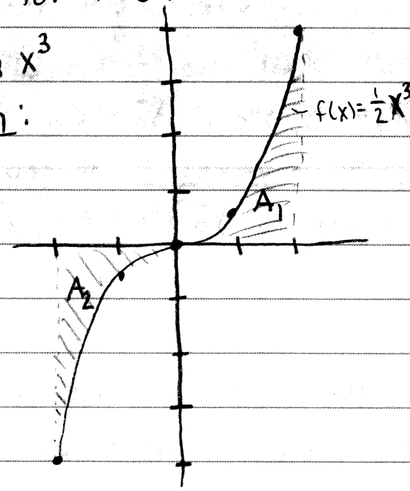
width = 2/5

$\int_{-1}^{1} (1-x^2)dx \approx (2/5)(f(-1) + f(-3/5) + f(-1/5) + f(1/5) + f(3/5))$
$= (2/5)(0 + (1-9/25) + (1-1/25) + (1-1/25) + (1-9/25))$
$= (2/5)(16/25 + 24/25 + 24/25 + 16/25)$
$= (2/5)(32/25 + 48/25) = (2/5)(80/25) = 2(16/25) \boxed{= 32/25}$

*See End for # 38 *

56. $\int_{-2}^{2} \frac{1}{2}x^3$

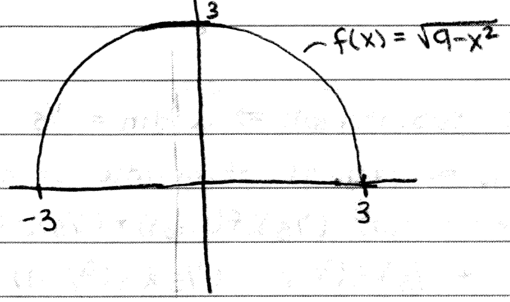
Graph:



$\int_{-2}^{2} \frac{1}{2}x^3 =$ [Area between x-axis and f(x) from x=0 to x=2] − [Area between x-axis and f(x) from x=-2 to x=0]

(In graph, this is "$A_1 - A_2$"

62.



Notice that this is a half circle of radius 3. So, $A=\frac{1}{2}\pi r^2$.
Thus, $A=\frac{1}{2}\pi(3)^2 \boxed{=\frac{9}{2}\pi}$

68. Given $\int_0^a x^2\,dx = \frac{1}{3}a^3$:

(a) $\int_0^2 \frac{1}{2}x^2\,dx = \frac{1}{2}\int_0^2 x^2\,dx = \frac{1}{2}\left(\frac{1}{3}(2)^3\right) = \frac{1}{6}(8) \boxed{= 4/3}$

(b) $\int_{-3}^0 2x^2\,dx = 2\int_{-3}^0 x^2\,dx = -2\int_0^{-3} x^2\,dx = -2\left(\frac{1}{3}(-3)^3\right)$
$= -\frac{2}{3}(-27) = 2(9) \boxed{= 18}$

(c) $\int_1^3 \frac{1}{3}x^2\,dx = \frac{1}{3}\int_1^3 x^2\,dx = \frac{1}{3}\left(\int_1^0 x^2\,dx + \int_0^3 x^2\,dx\right)$
$= \frac{1}{3}\left(-\int_0^1 x^2\,dx + \int_0^3 x^2\,dx\right) = \frac{1}{3}\left(-\frac{1}{3}(1)^3 + \frac{1}{3}(3)^3\right)$
$= \frac{1}{3}\left(-\frac{1}{3}+9\right) = \frac{1}{3}(26/3) \boxed{= 26/9}$

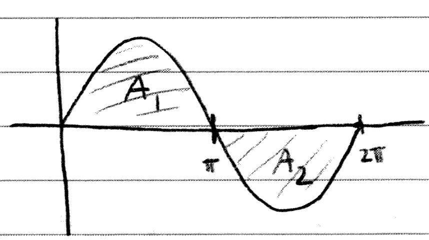
(d) $\int_1^1 3x^2\,dx \boxed{= 0}$

(e) $\int_{-2}^3 \frac{3}{2}x^2\,dx = \frac{3}{2}\int_{-2}^3 x^2\,dx = \frac{3}{2}\left(\int_{-2}^0 x^2\,dx + \int_0^3 x^2\,dx\right)$
$= \frac{3}{2}\left(-\int_0^{-2} x^2\,dx + \int_0^3 x^2\,dx\right) = \frac{3}{2}\left(-\frac{1}{3}(-2)^3 + \frac{1}{3}(3)^3\right)$
$= \frac{3}{2}\left(8/3 + 27/3\right) = \frac{1}{2}(8+27) = \frac{1}{2}(35) \boxed{= 35/2}$

(f) $\int_2^4 (x-2)^2\,dx = \int_0^2 x^2\,dx = \frac{1}{3}(2)^3 \boxed{= \frac{8}{3}}$

74. The area between the x-axis and $f(x)=x^2$ from $x=1$ to $x=2$ is the same as the area between the x-axis and f(x) from $x=0$ to $x=2$ minus the area between the x-axis and f(x) from $x=0$ to $x=1$, which is what $\int_1^2 x^2\,dx = \int_0^2 x^2\,dx - \int_0^1 x^2\,dx$ means. Assuming this, we can use Property 2 to rewrite this as $\int_1^2 x^2\,dx = \int_0^2 x^2\,dx - \left(-\int_1^0 x^2\,dx\right) = \int_0^2 x^2\,dx + \int_1^0 x^2\,dx$
This is exactly Property 5.

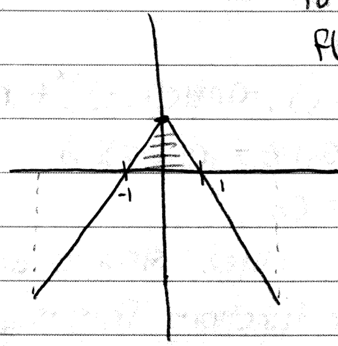
82.



$\int_0^a \sin x = 0$ when the area above the x-axis = area below the x-axis (and bound by $f(x)=\sin x$). This happens when $\boxed{a=2\pi}$
[a has to be in $(0,2\pi]$]
(using symmetry about $x=\pi$)

* SEE EXAMPLE 10 in SECTION 6.1 *

84. $\int_{-a}^{a} (1-|x|)\,dx = 0$ when the area above the x axis is equal to the area below the x-axis (bounded by $f(x) = 1-|x|$). The area above the x-axis is (using geometry) $A_+ = \frac{1}{2}(2)(1) = 1$



The area below the x-axis is (again, using geometry – two triangles, and symmetry)

$A_- = 2\left(\frac{1}{2}(a-1)|1-|a||\right) = (a-1)|(1-a)| = (a-1)^2$

$= a^2 - 2a + 1$

So, solving: $A_+ = A_-$

$1 = a^2 - 2a + 1$

$0 = a^2 - 2a$

$0 = a(a-2)$

$a = 0 \quad a = 2$

Reject $a=0$, since we have the condition that $a>0$. Therefore, $\boxed{a=2}$

Section 6.2

6. $y = \int_0^x \sqrt{1+u^2}\,du \quad x>0$

$dy/dx = \frac{d}{dx}\int_0^x \sqrt{1+u^2}\,du = \sqrt{1+x^2}$, since $\sqrt{1+u^2}$ is continuous at all $x>0$, so we may apply the FTC (I).

14. $y = \int_{\pi/4}^{x} \cos^2(u-3)\,du$

$dy/dx = \frac{d}{dx}\int_{\pi/4}^{x} \cos^2(u-3)\,du = \cos^2(x-3)$, since $\cos^2(x-3)$ is continuous everywhere, so we may apply the FTC (I).

16. $y = \int_0^{2x-1} (t^2-1)\,dt$

$dy/dx = \frac{d}{dx}\int_0^{2x-1} (t^2-1)\,dt = ((2x-1)^2-1)(2)$, using Leibniz's rule and FTC (I) since $((2x-1)^2-1)$ is continuous for all $x \in \mathbb{R}$.

24. $y = \int_2^{\ln x} e^{-t}\,dt \quad x>0$

$dy/dx = \frac{d}{dx}\int_2^{\ln x} e^{-t}\,dt = (e^{-\ln x})(1/x)$, using Leibniz's Rule since $e^{-\ln x}$ is continuous for all $x>0$.

32. $y = \int_{1+x^2}^{2} \tan t\,dt = -\int_2^{1+x^2} \tan t\,dt$

$dy/dx = -\frac{d}{dx}\int_2^{1+x^2} \tan t\,dt = -\tan(1+x^2)(2x)$, using Leibniz's Rule

* This problem actually cannot be done using Leibniz's Rule since $\tan t$ is not guaranteed to be continuous. But, if you could, this is how it would be done.

34. $y = \int_{-x}^{x} \tan u\,du \quad 0 < x < \pi/4$ Note: $\tan u$ is continuous for $0 < x < \pi/4$

Rewrite $y = \int_{-x}^{x} \tan u\,du = \int_0^x \tan u\,du + \int_{-x}^{0} \tan u\,du$

$= \int_0^x \tan u\,du - \int_0^{-x} \tan u\,du$

By Leibniz's Rule, $\frac{dy}{dx} = \frac{d}{dx}\int_{-x}^{x} \tan u\,du = \frac{d}{dx}\left(\int_0^x \tan u\,du - \int_0^{-x} \tan u\,du\right)$

$= \frac{d}{dx}\int_0^x \tan u\,du - \frac{d}{dx}\int_0^{-x} \tan u\,du = \tan x - \tan(-x)(-1)$

$= \tan x + \tan(-x) = \tan x - \tan x = 0.$

Note: $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$, since $\sin x$ is an odd function and $\cos x$ is an even function. This is NOT factoring out the negative from $\tan(-x)$.

38. $y = \int_{1+x^2}^{x^3-2x} \cos t\,dt = \int_0^{x^3-2x} \cos t\,dt + \int_{1+x^2}^{0} \cos t\,dt = \int_0^{x^3-2x} \cos t\,dt - \int_0^{1+x^2} \cos t\,dt$

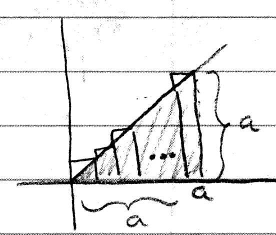
$\frac{dy}{dx} = \frac{d}{dx}\int_{1+x^2}^{x^3-2x} \cos t\,dt = \frac{d}{dx}\left(\int_0^{x^3-2x} \cos t\,dt - \int_0^{1+x^2} \cos t\,dt\right)$

$= \frac{d}{dx}\int_0^{x^3-2x} \cos t\,dt - \frac{d}{dx}\int_0^{1+x^2} \cos t\,dt = \cos(x^3-2x)(3x^2-2) - \cos(1+x^2)(2x)$

by Leibniz's Rule, since $\cos t$ is always continuous.

Section 6.1 # 38:

(a) $\int_0^a x\,dx$ = "Area of triangle" $= \frac{1}{2}(a)(a) = \boxed{\frac{1}{2}a^2}$



(b) There are "n subintervals":

$\int_0^a x\,dx \approx S_n = \sum_{k=1}^{n} \left(\frac{a}{n}\right)(f(x_k)) = \sum_{k=1}^{n}\left(\frac{a}{n}\right)\left(\frac{ka}{n}\right)$

$= \frac{a^2}{n^2}\sum_{k=1}^{n} k = \frac{a^2}{n^2}\left(\frac{n(n+1)}{2}\right) = \frac{a^2(n+1)}{2n} = \boxed{\frac{a^2 n + a^2}{2n}}$

* Notice $\lim_{n\to\infty} S_n = \frac{a^2}{2} = \frac{1}{2}a^2$. Compare this to part (a).