HOMEWORK #1 Due w/o 9/14

Section 1.1 Covered, but no HW Problems

Section 1.2

2. rng $f(x) = [0, 1]$

4. rng $f(x) = [0, 1/4)$

6. (a) By definition, $2|x-1| = \begin{cases} 2(x-1) & \text{if } x-1 \geq 0 \Rightarrow x \geq 1 \\ -2(x-1) & \text{if } x-1 \leq 0 \Rightarrow x \leq 1 \end{cases}$

Simplifying, $2|x-1| = \begin{cases} 2(x-1) & \text{if } x \geq 1 \\ 2(1-x) & \text{if } x \leq 1 \end{cases}$

(b) Expanding $g(x) = 2|x-1| = \begin{cases} 2(x-1) & 1 \leq x \leq 2 \\ 2(1-x) & 0 \leq x \leq 1 \end{cases}$

$= \begin{cases} 2-2x & \text{if } 0 \leq x \leq 1 \\ 2x-2 & \text{if } 1 \leq x \leq 2 \end{cases}$ $\therefore$ Yes, they are equal

16. $f(x) = 1/x+1 \quad (x \neq -1)$ $\qquad g(x) = 2x^2 \quad (x \in \mathbb{R})$

(a) $(f \circ g)(x) = f(g(x)) = f(2x^2) = \frac{1}{2x^2+1} \quad (x \in \mathbb{R})$

(b) $(g \circ f)(x) = g(f(x)) = g(1/x+1) = 2\left(\frac{1}{x+1}\right)^2 = 2/(x+1)^2 \quad (x \neq -1)$

26. $0 \leq x \leq 1$ mult. By x^{n-1}, and lower

$x^n \leq x^{n-1} \leq x^{n-2} \leq \cdots \leq 1$, Thus, true $\forall\, m \leq n$

$x \geq 1$ mult. by x^{n-1} and lower.

$x^n \geq x^{n-1} \geq x^{n-2} \geq \cdots \geq 1$

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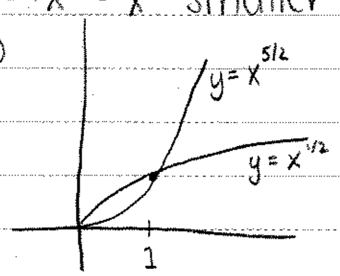
38. $y = x^{-n}$, $y = x^{-m}$ $\quad m \leq n$

$x^{-n} = x^{-m}$ Intersect at 1

$y = x^{-n} = \frac{1}{x^n}$ smaller if $x < 1$

$y = x^{-m} = \frac{1}{x^m}$ smaller if $x > 1$

52. (a)



(b) $x \leq 1 \Rightarrow x^{3/2} \leq x^{1/2}$

$\Rightarrow x^{5/2} \leq x^{3/2}$

$\therefore x^{5/2} \leq x^{1/2}$

(c) $x \geq 1 \Rightarrow x^{3/2} \geq x^{1/2}$

$\Rightarrow x^{5/2} \geq x^{3/2}$

$\therefore x^{5/2} \geq x^{1/2}$

58. $N(t) = 40 \cdot 2^t \quad t \geq 0$

(a) $N(0) = 40 \cdot 2^0 = 40 \cdot 1 = 40.$

(b) Note that $2^t = e^{\ln 2^t} = e^{t \ln 2}$

$\therefore N(t) = 40 e^{t \ln 2} \quad t \geq 0.$

(c) $N(t) = 1000 = 40 e^{t \ln 2}$

$25 = e^{t \ln 2}$

$\ln 25 = \ln e^{t \ln 2}$

$\ln 25 = t \ln 2$

$\ln 25 / \ln 2 = t.$

60. Half Life $= 5730$ yrs $= T_h$

$t = 0 \Rightarrow W_0 = 20 \mu g.$

(a) Notice that $10 \mu g$ is half of W_0. Thus, it takes one half life $= 5730$ yrs.

(b) $5 \mu g$s is one half life from part (a). Thus, it will take $5730 + 5730 = 11460$ yrs.

64. $.35 = e^{-\lambda t}$

$\ln .35 = \ln e^{-\lambda t}$

$\ln .35 = -\lambda t$

$\ln\left(\frac{1}{.35}\right) = \lambda t$

$\lambda = \ln 2 / T_h = \ln 2 / 5730$

$\ln\left(\frac{1}{.35}\right) = \left(\ln 2 / 5730\right)(t)$

$5730 \left(\frac{\ln(1/.35)}{\ln(2)}\right) = t$

≈ 8678 years.

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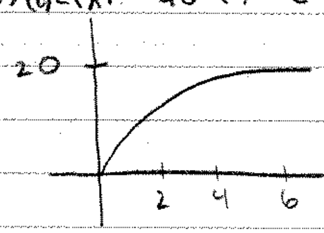
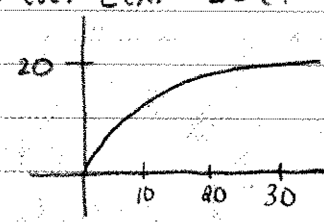
68. (a) (i) $L(x) = 20(1 - e^{-x})$



(b) (ii) $L(x) = 20(1 - e^{-.1x})$



(b) $.9(L_\infty) = L_\infty(1 - e^{-x})$

$.9 = 1 - e^{-x}$

$e^{-x} = .1$

$-x = \ln .1$

$x = \ln(10)$

$.99(L_\infty) = L_\infty(1 - e^{-x})$

$.99 = 1 - e^{-x}$

$e^{-x} = .01$

$-x = \ln .01$

$x = \ln(100)$

If they could: $L_\infty = L_\infty(1 - e^{-x}) \Rightarrow 1 = 1 - e^{-x} \Rightarrow e^{-x} = 0$ ↯.

No, they don't. This is an upper bound / asymptote.

(c) • When $k = 1$

• As k decreases towards zero, the cure increases more slowly. $k \nearrow \Rightarrow$ curve approaches L_∞ faster.

70. (a) Suppose $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbb{R}$.

Thus, $x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1^3 = x_2^3$ → since x^3 is one to one (See figure 1.27)

Since the cube root is unique, $x_1 = x_2$.

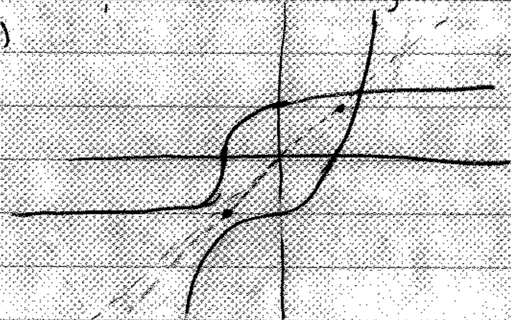
*or use graph / horizontal line test. ⊠

$f(x) = x^3 - 1 \Rightarrow y = x^3 - 1$

$x = y^3 - 1 \Rightarrow x + 1 = y^3 \Rightarrow y = \sqrt[3]{x+1}$

$\therefore f^{-1}(x) = \sqrt[3]{x+1}$, $x \in \mathbb{R}$.

(b)



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74. $f(x) = 2x + 1$ $x \in \mathbb{R}$.

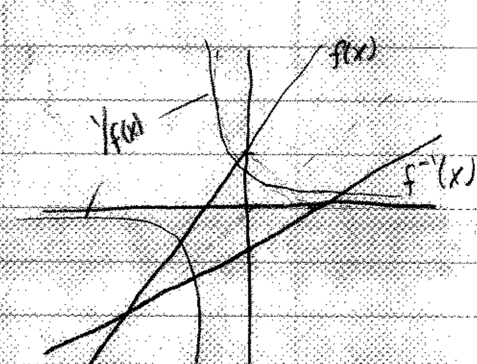
$1/f(x) = \frac{1}{2x+1}$

$f^{-1}(x) = \frac{x-1}{2}$

$x = 2y + 1$

$x - 1 = 2y$

$y = \frac{x-1}{2}$



88. $y = a^x$

$\log_a y = x$

$\ln y / \ln a = x$

$\ln y = x \ln a$

$y = e^{x \ln a}$ $\mu = -\ln a$

90. (a) $S = 5 \Rightarrow H = -(p_1 \ln p_1 + p_2 \ln p_2 + p_3 \ln p_3 + p_4 \ln p_4 + p_5 \ln p_5)$

$p = 1/5$, Equal dist. $\left\{ \begin{array}{l} = -(p\ln p + p\ln p + p\ln p + p\ln p + p\ln p) \\ = -5p\ln p = -5(1/5)\ln(1/5) = -\ln(5)^{-1} = \ln 5 \end{array} \right.$

(b) $S = 10 \Rightarrow H = -10p\ln p = -10(1/10)\ln(1/(10)) = -\ln(10^{-1}) = \ln(10)$

(c) $H/\ln 5 = \ln 5/\ln 5 = 1$

$H/\ln 10 = \ln 10/\ln 10 = 1$

(d) $S = N \Rightarrow H = -(p_1 \ln p_1 + \dots + p_N \ln p_N) = -(N p \ln p) = -(N(1/N)\ln(1/N))$

$= -\ln N^{-1} = \ln N \Rightarrow \ln N/\ln N = 1.$ ✓

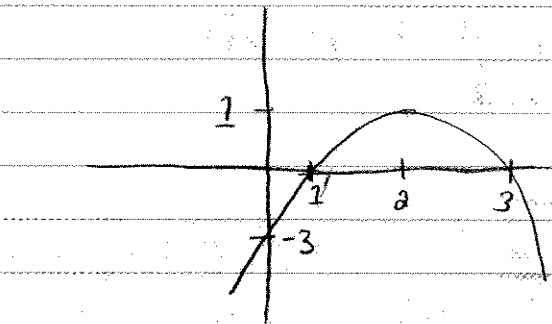
98. $f(x) = -2\sin(x/2)$ $x \in \mathbb{R}$ $|a| = 2$; $|k| = \frac{1}{2}$

Amplitude $= 2$, Period $= 2\pi/|k| = 2\pi/\frac{1}{2} = 4\pi$

GRADE 104. $f(x) = -\frac{2}{3}\cos\left(\frac{3x}{\pi}\right)$, $x \in \mathbb{R}$ $|a| = 2/3$, $|k| = 3/\pi$

Amplitude $= 2/3$, Period $= 2\pi / 3/\pi = \frac{2}{3}\pi^2$

Section 1.3 2.



$y = x^2 \rightarrow y = (x-2)^2$ [shift right 2]

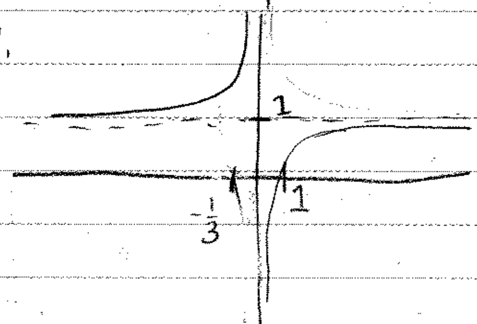
$\rightarrow y = -(x-2)^2$ [Reflect on x-axis]

$\rightarrow y = -(x-2)^2 + 1$ [Shift up 1]

$\cancel{1 = (x-2)^2}$

$\cancel{\pm 1 = x - 2}$ $x = 1, 3$

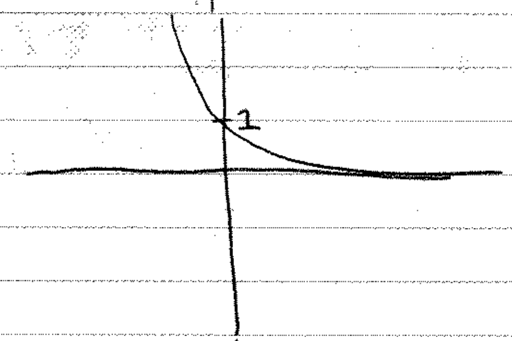
8.



$y = 1/x \rightarrow y = -1/x$ [Reflect on x-axis]

$\rightarrow y = 1 - 1/x$ [Shift up 1

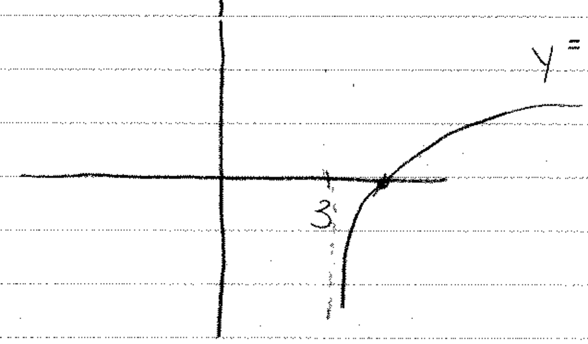
12.



$y = \exp(x) \rightarrow y = \exp(-x)$

[Reflect on y-axis]

GRADE 16.



$y = \ln(x) \rightarrow y = \ln(x-3)$

[Shift Right 3]