Name: _____

110.106 CALCULUS I (Biology & Social Sciences)
FALL 2009
MIDTERM EXAMINATION
October 12, 2009

Instructions: The exam is **8** pages long, including this title page. The number of points each problem is worth is listed after the problem number. The exam totals to one hundred points. For each item, please **show your work** or **explain** how you reached your solution. Please do all the work you wish graded on the exam. Good luck !!

PLEASE DO NOT WRITE ON THIS TABLE !!

Problem	Score	Points for the Problem
1		20
2		20
3		8
4		8
5		16
6		18
7		10
TOTAL		100

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: _____ **Date:**_____

Question 1. [20 points] Evaluate the following:

(a) $\lim_{x \to -\infty} \dfrac{3x - 4x^3}{2x^3 + 6x^2}$.

(b) $\lim_{x \to 0} \dfrac{\sin(4x)}{-x}$.

Question 2. [20 points] Evaluate the following:

(a) $\lim_{n\to\infty} a_n$, For $a_n = \dfrac{1+\sin n}{n^2}$.

(b) Assuming $\lim_{n\to\infty} b_n$ exists for the recursively defined sequence

$$b_{n+1} = \frac{5}{2} b_n(1 - b_n),$$

What are the only possible values for the limit (that is, what are the fixed points of $\{b_n\}$)?

Question 3. [8 points] Find a value of the constant c so that the function $f(x)$ is continuous at $x = 2$, where

$$f(x) = \begin{cases} x^2 + cx + 1 & x < 2 \\ \frac{8}{x^2} & x \geq 2 \end{cases} .$$

Question 4. [8 points] Show $h(x) = x^7 + 5x^3 - 1$ has a root in the interval $[0, 1]$.

Question 5. [16 points] Let $f(x) = \dfrac{\sqrt{x^2 + 6x}}{3x}$. Do the following:

(a) Find the domain of f.

(b) Calculate $f'(x)$.

Question 6. [18 points] Let $g(x) = 2x^2 - 3$. Do the following:

(a) Use the definition of the derivative to show that $g'(2) = 8$.

(b) Find the equation of the line tangent to $g(x)$ at $x = 2$.

(c) For $f(x) = \dfrac{1}{\sqrt{x}}$, find $(g \circ f)(x)$ and specify its domain.

Question 7. [10 points] Do exactly ONE of the following:

(a) Find the slope of the line tangent to the curve given by the equation $y^2 = x^2 + xy$ at the point $(-1, 0)$.

(b) The volume of a spherical balloon is $V = \frac{4}{3}\pi r^3$ where the radius r is measured in centimeters. If the volume is expanding at a constant rate of 100 $\frac{cm^3}{sec}$, how fast is the radius expanding when the radius is exactly 10 cm?