Mathematic 405, Fall 2015: Assignment #6

Due: **Wednesday, March 25th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Let $f : D \to \mathbb{R}$ be differentiable at $x_0 \in D$ and $g(x) = f(x_0) + m(x - x_0)$ be an affine function.

a) Show that $f - g = O(|x - x_0|), x \to x_0$.
b) Show $f - g = o(|x - x_0|), x \to x_0$ if and only if $m = f'(x_0)$.
c) If f is *not* differentiable at x_0, is it true that $f - g \neq O(|x - x_0|), x \to x_0$.

Problem #2. pg. 152 # 2.

Problem #3. pg. 152 # 3.

Problem #4. pg. 152 # 6.

Problem #5. pg. 152 # 9. Here we say $f_M \to h$ as $M \to \infty$ if, for each x for which this makes sense, $\lim_{M\to\infty} f_M(x) = h(x)$.

Problem #6. pg. 163 # 2.

Problem #7. pg. 163 # 10.

Problem #8. Let $I = (a, b)$ be an interval. Show that if $f : I \to \mathbb{R}$ is differentiable and $|f'| < M < \infty$, then f is Lipschitz with Lipschitz constant M. Give an example to show that this is not possible without the uniform bound (i.e. give a differentiable function which is not Lipschitz.) Hint: use the Mean Value Theorem.