

Mathematic 405, Fall 2015: Assignment #4

Due: **Wednesday, March 4th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Show that a set $X \subset \mathbb{R}$ is closed if and only if every convergent sequence $\{x_n\}$ with $x_n \in X$ has $\lim_{n \rightarrow \infty} x_n \in X$.

Problem #2. Let C be the Cantor set. Show that C has no interior points. (Hint: show that any number in $[0, 1]$ can be well approximated by a rational number of the form $i3^{-n}$ for $0 \leq i \leq 3^n$).

Problem #3. p. 98 # 7

Problem #4. Let $A_n \subset \mathbb{R}$ be a nested set of subsets – i.e., $A_{n+1} \subset A_n$. Show that if each of the A_n are compact, then $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$.

Problem #5. p. 106 # 1

Problem #6. p. 106 # 6

Problem #7. p. 125 # 2

Problem #8. p. 125 # 15