Mathematic 405, Fall 2019: Assignment #2

Due: **Wednesday, September 18th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Let F be an ordered field. Prove directly from the axioms of that if $x, y, z \in F$, $y < z$ and $x < 0$, then $xy > xz$.

Problem #2. Let S be an ordered set. Suppose that $A \subset S$ is such that $s = \sup(A) \in S$ exists, but $s \notin A$. Show that A must contain a countably infinite subset.

Problem #3. Let $S = \mathbb{N} \cup \{\infty\}$. Make this an ordered set by making $n < \infty$ for every $n \in \mathbb{N} \subset S$ and otherwise using the usual order on $\mathbb{N}$. Show that every $E \subset S$ is bounded and $\sup(S)$ and $\inf(S)$ both exist.

Problem #4. Let $x, y \in \mathbb{R}$ satisfy $x^2 + y^2 = 0$ show that $x = y = 0$.

Problem #5. Use induction to show that for any $x \in \mathbb{R}$ if $1 + x > 0$, then $(1 + x)^n \geq 1 + nx$.

Problem #6. Show that as subsets of $\mathbb{R}$, $\sup\{x \in \mathbb{Q} : x^2 < 5\} = \sup\{x \in \mathbb{R} : x^2 < 5\}$.

Problem #7. For $x, y \in \mathbb{R}$ show that

$$\max\{x, y\} = \frac{1}{2}\left(x + y + |x - y|\right) \text{ and } \min\{x, y\} = \frac{1}{2}\left(x + y - |x - y|\right).$$

Problem #8. Let A be a non-empty set and $f, g : D \to \mathbb{R}$ be two bounded functions. Show that

$$\sup_{x \in D}\left(f(x) + g(x)\right) \leq \sup_{x \in D} f(x) + \sup_{x \in D} g(x).$$

Give an example to show the inequality can be strict.