Mathematic 405, Fall 2019: Assignment #1

Due: **Wednesday, September 11th**

Instructions: Please ensure that your answers are legible. Also make sure that sufficient steps are shown. Page numbers refer to the course text.

Problem #1. Use mathematical induction to show that $n^3 + 5n$ is divisible by 6 for all $n \in \mathbb{N}$.

Problem #2. Show that there is no $x \in \mathbb{Q}$ so that $x^2 = 6$.

Problem #3. For a given number $n \in \mathbb{N}$ with $n > 1$, use the well ordering principle of $\mathbb{N}$ to show that n has at least one prime factor. (Hint: Consider F_n the set of factors of n which are greater than 1).

Problem #4. For any set A, let $2^A = \{f : f : A \to \mathbb{N}_2\}$. Show that 2^A is in bijection with $P(A)$, the power set of A. Here, $\mathbb{N}_N = \{1, 2, \ldots, N\} \subset \mathbb{N}$. (Hint: Construct natural maps between the two sets).

Problem #5. Show that every subset of $\mathbb{N}$ is either finite or countable (Hint: Use the well ordering property of $\mathbb{N}$). Conclude that every subset of a countable set is finite or countable.

Problem #6. Let A_n be collection of countable sets, where $n \in \mathbb{N}$. Show that

$$A = \bigcup_{n \in \mathbb{N}} A_n = \{x : \exists n \in \mathbb{N} \text{ so that } x \in A_n\}$$

is countable.

Problem #7.

a) Show that if both A and B are countable, then so is $A \times B$.
b) Show that $\mathbb{Q}$, the set of rational numbers, is countable.

Problem #8. Determine whether, $P_{finite}(\mathbb{N})$ the set of all finite subsets of $\mathbb{N}$, is countable.