Mathematic 108, Fall 2015: Assignment #7

Due: **In your assigned section, either Tues., Oct. 27th or Thurs., Oct. 29th**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Suppose f is a function with f'' continuous on the open interval I and so that f has a local maximum at both $x = a$ and $x = b$ for $a, b \in I$ with $a < b$. Explain why there must be value $c \in (a, b)$ so $f''(c) \geq 0$. (Hint: If $f''(x) < 0$ for all $x \in (a, b)$, then any critical number of f in (a, b) is a local maximum).

Problem #2. Give an example of a continuous function with domain $[-1, 1]$ with a local maximum, but no local minimum.

Problem #3. Give an example of a function f with continuous second derivative for which f'' is zero at some point and whose graph does not have an inflection point.

Problem #4. Determine whether the following functions have an absolute maximum value and absolute minimum value on the given domain. If it does determine the value.

a) $f(x) = \frac{1}{1+e^{-x^2}}$ on $D = (-\infty, \infty)$.
b) $f(x) = \tan^{-1}(x) + \frac{1}{1+x^2}$ on $D = (-\infty, \infty)$.
c) $f(x) = x - \sqrt{x^2 + 3}$ on $D = [0, \infty)$.

Problem #5. Let $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ and $g(x) = \sin(x)$.

a) Use the limit laws to show that $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$ (Hint: Consider $\lim_{x\to 0} \frac{f(x)/x}{g(x)/x}$).
b) Determine $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ how do you reconcile this with a) and L'Hospital's Rule.

Problem #6. Use L'Hospital's Rule to evaluate the following limits

a) $\lim_{x\to 0} \frac{\arcsin(2x)}{x}$.
b) $\lim x \to 0^+ \left(\frac{1}{x} - \frac{1}{\arctan(x)}\right)$.
c) $\lim_{x\to 0^+} (1 + \sin(2x))^{1/x}$.

Problem #7. Suppose f is differentiable, $f(3) = 1$ and $f'(3) = -2$. Evaluate $\lim_{x\to 0} \frac{f(3+x)-f(3-4x)}{x}$.

Problem #8. Suppose g is differentiable, $g(2) = 0$ and $g'(2) = 3$. Evaluate $\lim_{x\to 1} \frac{g(1+x)+g(4-2x)}{x}$.

Problem #9. Use the methods of Section 4.5 to sketch the following curves

a) $y = \frac{(x-2)^2}{x^2+1}$
b) $y = x - \sin(2x)$
c) $y = \sqrt{1 + x^2} - x$

Problem #10. Consider the family of polynomials $P_c(x) = x^3 + 3cx^2 + 3x$.

a) Determine the values of c so that P_c has both a local maximum and a local minimum.
b) Sketch the graph of $y = P_c(x)$ for a value c for which c has both a local maximum and minimum and sketch the graph for a value c for which it does not.

Book Problems.

a) Section 4.4: #4, #32, #44, #76, #88
b) Section 4.5: #2, #12, #50, #72
c) Section 4.6: #28