Mathematic 108, Fall 2015: Assignment #6

Due: **In your assigned section, either Tues., Oct. 20th or Thurs., Oct. 22nd**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Determine the critical numbers of the following functions

a) $f(x) = 2x^3 + x^2 + 2x - 4$.
b) $f(x) = |x-1| + x^2$.

Problem #2. Find the absolute maximum and minimum values of the given function on the given interval

a) $f(x) = -1 + 36x - 3x^3$, $[-3, 1]$.
b) $f(x) = e^x - x$, $-1 \leq x \leq 1$.
c) $f(x) = x + \cos(x)$, $[0, 2\pi]$.

Problem #3. Explain why the function $f(x) = e^{-2x} - x^{101} - 2$ has no local maxima or minima.

Problem #4. Let $f(x) = 1 - x^{4/5}$. Show that $f(-1) = f(1)$, but there is no value c in $(-1, 1)$ so that $f'(c) = 0$. Why does this not contradict Rolle's theorem.

Problem #5. Show that the equation $e^{2x} + e^x = -x$ has exactly one real root.

Problem #6. Use the Mean Value Theorem to show that for all x, y

$$|\arctan(x) - \arctan(y)| \leq |x - y|.$$

Problem #7. Use the Mean Value Theorem to show that if $y = f(x)$ has domain $(-\infty, \infty)$ and satisfies the ODE $y' + 3y = 0$ (i.e. $f'(x) + 3f(x) = 0$ for all x in the domain of f), then $f(x) = Ce^{-3x}$ for some constant C. What happens if $y = f(x)$ satisfies the same ODE but has domain $(-\infty, -1) \cup (1, \infty)$? (Hint: consider the derivative of $g(x) = e^{3x} f(x)$).

Problem #8. Determine the intervals of increase and decrease and intervals of concavity for the following functions.

a) $f(x) = x^2 e^{-x}$.
b) $f(x) = \cos^2(x) + 2\sin(x)$ and $-2\pi \leq x \leq 2\pi$.

Problem #9. For what values of c is the function $f(x) = cx + \frac{1}{x^2+3}$ decreasing on $(-\infty, \infty)$? (Hint: try to determine the maximum value of $f'(x)$).

Problem #10. Suppose $f(x)$ satisfies $f''(x) > 0$ for all $x \in (-\infty, \infty)$. Explain why $f(x) \geq f'(a)(x-a) + f(a)$ for all $x \in (-\infty, \infty)$. What does this mean geometrically about the relationship between the graph of $y = f(x)$ and the graph of its tangent line at $(a, f(a))$?

Book Problems.

a) Section 4.1: #4, #38, #56
b) Section 4.2: #6, #18, #22, #26
c) Section 4.3: #8, #24, #52