Mathematic 108, Fall 2015: Assignment #2

Due: **In your assigned section, either Tues., Sep. 15th or Thurs., Sep. 17th.**

Instructions: Please ensure your name, your TA's name and your section number appear on the first page. Also that your answers are legible and all pages are stapled. Page numbers refer to the course text.

Problem #1. Simplify $\cos(2\arcsin(\frac{1}{2}x))$.

Problem #2. Consider the function $f(x) = \begin{cases} 2(x-1)^2 - 1 & x \leq 1 \\ x^3 & 2 \leq x < 3. \end{cases}$

a) Determine the equation of the secant line between $(0, f(0))$ and $(2, f(2))$.
b) Let $m(x)$ be function whose value at x is the slope of the secant line between $(0, f(0))$ and $(x, f(x))$. Determine $m(x)$ and its domain.
c) Calculate $\lim_{x\to 0} m(x)$.

Problem #3. Determine the domain and the values near (we defined this in class) which the following functions are defined.

a) $f(x) = \arcsin(x^2)$.
b) $f(x) = \begin{cases} -x & -2 \leq x \leq 0 \\ \cos(x) & x > 0. \end{cases}$

Problem #4. Determine the following infinite limits

a) $\lim_{x\to 0^+} \ln\left(\frac{1}{x}\right)$.
b) $\lim_{x\to 0^+} \left(\frac{1}{\sqrt{x}} - \ln(x)\right)$.
c) $\lim_{x\to(-\frac{\pi}{2})^-} x\tan(x)$

Problem #5. Evaluate the limit, if it exists.

a) $\lim_{x\to -1} \frac{x+1}{x^3+1}$.
b) $\lim_{x\to 1}(x^2-1)(x^2+1)$.
c) $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{|x|}$.

Problem #6. Let $f(x) = \begin{cases} 2 - x^2 - c & x < -1 \\ \sqrt{x+c} & x > -1 \\ 10 & x = -1. \end{cases}$ Determine values c so that $\lim_{x\to -1} f(x)$ exists.

Problem #7. If $f(x)$ is defined near $x = -1$ and $x^3 - x + 2 \leq f(x) \leq 4 + 2x$, then calculate $\lim_{x\to -1} f(x)$.

Problem #8. Use the ϵ, δ definition of limit to show that $\lim_{x\to 1} \frac{1}{x} = 1$.

Problem #9. Use the precise definition of infinite limit to show that $\lim_{x\to -1} \frac{1}{(x^2-1)^2} = \infty$.

Problem #10. You wish to mill an inanimate carbon rod (i.e., a cylinder) of cross-sectional area 300 cm^2.

a) You want to get the desired area within ± 1 cm^2 – i.e., the error tolerance for the area ± 1 cm^2. What is the ideal radius of the rod? How close do you need to get to this value? I.e., what is the error tolerance for the radius?
b) Interpret this question in terms of the ϵ, δ definition of limit. What are the independent and dependent variables? The function? The ϵ and the δ?

Book Problems.

a) Section 2.1: # 2
b) Section 2.2: # 4, # 16
c) Section 2.3: #2, #6, # 10, # 14, # 50.
d) Section 2.4: #2, #38