Calculus 108, Fall 2014 : Pre-Midterm Practice

Johns Hopkins University

Problem 1

Let $f(x)$ be a function defined as

$$f(x) = 3\,|x| - 5.$$

- (A) Plot the graph of $f(x)$ in the x-y plane and mark the intercepts clearly.
- (B) At which points is the function $\log f(x)$ well defined.

Problem 2

Compute the following limits. Briefly justify the steps you take.

- (A) $\lim_{x\to 2} \dfrac{1-\frac{2}{x}}{x^2-4}$
- (B) $\lim_{x\to 1} \dfrac{x^2-3x-2}{x^2+x-1}$
- (C) $\lim_{x\to \infty} \dfrac{e^x - e^{-2x}}{e^{3x}+e^{-4x}}$
- (D) $\lim_{x\to 0} \dfrac{\sin 5x}{\sin 4x}$
- (E) $\lim_{x\to 0} \dfrac{\sin x^2}{x}$

Problem 3

Find all asymptotes (horizontal and vertical) for the function

$$f(x) = \frac{\sqrt{4x^2+1}}{x+1}.$$

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Problem 4

Let c and d be constants, and define

$$f(x) = \begin{cases} cx - 1, & x \in (-\infty, -1) \\ c^2x^2 + cx + d, & x \in [-1, 0] \\ \dfrac{\sin(1/x)}{\ln(1/x)}, & x \in (0, \infty). \end{cases}$$

Do there exist $c, d \in \mathbb{R}$ such that f is continuous on all of $\mathbb{R}$? If so, find them. If not, explain why.

Problem 5

Find the derivative of the following functions, wherever it exists

- (A) $x^5 \sqrt[3]{x^3 - 8}$
- (B) $\sqrt{x + \sqrt{x}}$
- (C) $\dfrac{\log \sin x}{\log \cos x}$
- (D) $x^3 \tan^{-1}(x^3)$

Problem 6

Use logarithmic differentiation to find the derivative of the following functions. Please note that log stands for $\log_e$

- (A) $\dfrac{(x^2 + 2)^2}{(x^4 + 4)^4}$
- (B) $(\sin x)^{\log x}$
- (C) $(\log x)^{\cos x}$

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