1. Evaluate the following:

(a)

$$\int \arcsin x \, dx$$

(b)

$$\int \frac{e^{2x}\,dx}{e^{2x}-1}$$

(c)

$$\frac{d}{dx}\int_{\arcsin x}^{\pi/2} \ln(\sin t)\, dt$$

2. The volume V (in cubic inches) and pressure p (in pounds per square inch) of the air inside a balloon satisfy the equation $pV = 1000$. At what rate is the volume of the balloon changing if the pressure is currently 100 lb/in^2 and is dropping at the rate of 2 lb/in^2 per second?

3. Solve the initial value problem:

$$\frac{dy}{dx} = \frac{\sqrt{x}}{1+x}, \qquad y = 2 \text{ when } x = 1.$$

4. (a) Find the first (linear) and second (quadratic) Taylor polynomials for the function $f(x) = \sqrt{x}$ centered about the point 1.

(b) Use part (a) to find the first- and second-order approximations to the exact value of $\sqrt{101}$. [Hint: $\sqrt{101} = 10\sqrt{1.01}$.]

5. Decide if the improper integral

$$\int_1^\infty \frac{x\,dx}{x^4+1}$$

converges or diverges. Justify your answer.

6. A rectangle with sides parallel to the coordinate axes has one vertex at the origin, one on the positive x-axis, one on the positive y-axis, and its fourth vertex on the line $y = 100 - 2x$ (see the figure). What is the maximum possible area of such rectangle? What are its dimensions?

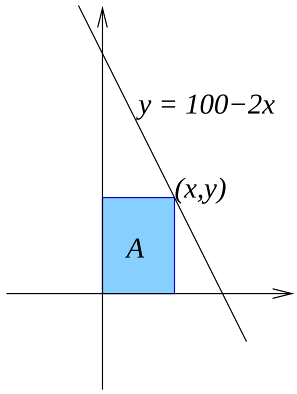

Figure 1: The rectangle inscribed under $y = 100 - 2x$.

7. A spherical container of radius 10 feet is partly filled with water (see figure). The depth of the water remaining in the container is 5 feet. Would you intuitively expect that the container is exactly one-quarter full, or less than one-quarter full, or more? Find the exact volume of the water inside (for reference, the capacity of the full container is $\frac{4000}{3}\pi$ cubic feet.)

   [Hint: Find the volume of the solid of revolution obtained by revolving the shaded circular wedge of the semi-circle $y = \sqrt{100 - x^2}$ around the x-axis.]

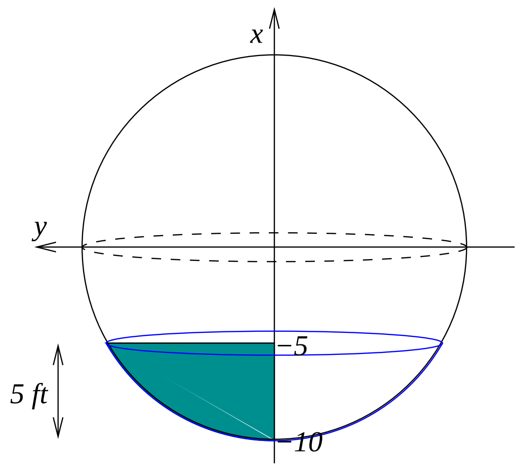

Figure 2: Cross-section of the water container.

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8. The graph of $(x^2 + y^2 + y)^2 = x^2 + y^2$ is a cardioid ("heart-shaped"). Find the equation of the tangent line to the cardioid passing through the point (1,0). (Hint: use implicit differentiation.)

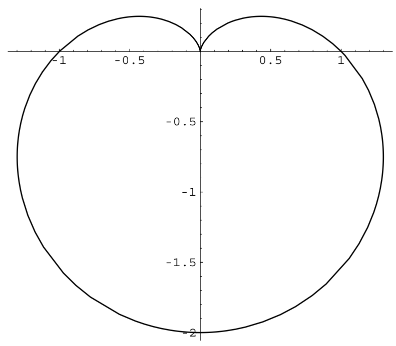

Figure 3: The cardioid $(x^2 + y^2 + y)^2 = x^2 + y^2$.