1. Compute the following limits.

   (a) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin 2x}$.

   (b) $\lim_{x \to 0+} x^x$.

   (c) $\lim_{x \to 1} \dfrac{1 - x + \ln x}{1 + \cos \pi x}$.

2. Consider the function $f(x) = \dfrac{x^3 - 4}{x^2}$.

   (a) Compute $f'(x)$ and analyze the regions where $f(x)$ is increasing or decreasing.

   (b) Compute $f''(x)$ and analyze the concavity of $f(x)$.

Problem 2. (continued)

(c) Find all asymptotes (horizontal, vertical or oblique) to the graph of $y = f(x)$.

(d) Sketch the graph of $y = f(x)$.

3. A wooden beam with rectangular cross section must be cut from a log with a circular cross section of diameter 2 feet. The strength of the beam is the product of its width w with the *square* of its height h. Find the optimal way to cut the beam from the log to maximize its resistance (aka, find the height and width of the most resistant beam.)

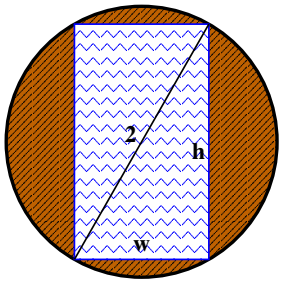

Figure 1: Cross section of the log and the beam to be cut from it.

4. Find the area of the "boomerang" contained between the graphs of $f(x) = \frac{x}{3\pi}(3\pi - x)$ and $g(x) = -\sin x$ for $0 \leq x \leq 3\pi$.

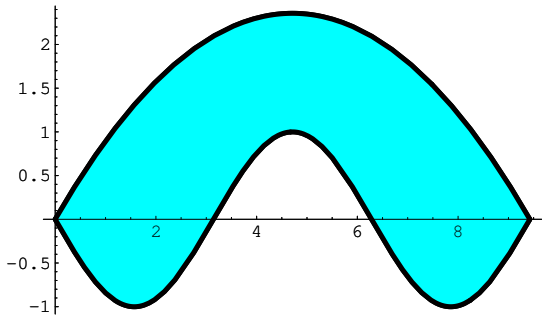

Figure 2: The "boomerang" $0 \leq x \leq 3\pi, g(x) \leq y \leq f(x)$.