1. Find the following limits. Show your work!

   (a) (10 points) $\lim_{u \to -3} \sqrt{u^3 + 3u^2 + 1}$

   (b) (10 points) $\lim_{x \to 1} \dfrac{1 - x^2}{x^2 + x - 2}$

   (c) (10 points) $\lim_{x \to +\infty} \dfrac{\exp(-x) + 3 - x^2}{4x^2 - 7}$

2. Consider the function

$$f(x) = \begin{cases} \exp(x), & \text{if } x \geq 0; \\ x + k, & \text{if } x < 0, \end{cases} \tag{1}$$

where k is a constant.

(a) (10 points) Graph the function assuming $k = 2$.

(b) (10 points) Find all possible values of k for which the function is continuous. *Justify your answer!*

3. Compute and simplify all answers as much as possible.

(a) (10 points) $\dfrac{d^2}{dx^2}\left(3x^4 - \sqrt{x^2+1} + \dfrac{3}{2x-1}\right)$.

(b) (10 points) $\dfrac{d}{dx}\ln(\sec x + \tan x)$.

(c) (10 points) $\dfrac{d}{dx}\ln\sqrt{\dfrac{x-1}{x+1}}$.

4. Consider the function

$$f(x) = \frac{x^2}{x-1}. \tag{2}$$

(a) (5 points) Show that the line $x = 1$ is an asymptote to the graph of $y = f(x)$ by computing $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$.

(b) (5 points) Show that the line $y = x + 1$ is another asymptote by computing $\lim_{x \to +\infty} [f(x) - (x+1)]$ and $\lim_{x \to -\infty} [f(x) - (x+1)]$.

(c) (5 points) Sketch the graph of $y = f(x)$. (Hint: it's a hyperbola.)

(d) (5 points) Find the equation of the tangent line to the graph passing through the point $(2, 4)$ and draw this line in the graph above.