

**Calculus I - 110.106**  
FINAL EXAM - Spring 2000

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**Please show all work. Partial credit will be given (20 points each).**

1. [20] Find  $\frac{dy}{dx}$  if:

a)  $y = \sin(x^4)$

c)  $y = \frac{x^2 + 1}{x^2 - a}$

b)  $y = \arctan(x^2)$

d)  $y = x(\ln x)^2$ .

2. [20] Calculate the following limits:

a)  $\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1}$

c)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$

b)  $\lim_{x \rightarrow 1^+} \frac{x^3}{x^3 - 1}$

d)  $\lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h}$ .

3. [20] Find the slope of the tangent line to the circle (of radius one) passing through the point P (as indicated above).

4. [20] Sketch the graph of the function  $y = 2x^3 - 6x + 7$ .

a) Indicate the relative maxima and minima.

b) Where is the function concave up?

c) Where is the function concave down?

5. [20] Show that among all rectangles with perimeter  $p$ , the square has the maximum area.
6. [20] A spherical balloon is inflated so that its volume is increasing at a rate of  $10 \text{ ft}^3/\text{min}$ . How fast is the radius increasing when the radius is 2 ft?

7. [20] Solve for  $x$ :

a)  $\log_3(1 + x^2) = 2$

c)  $\log_4(16^{3x}) = 36$

b)  $\arctan x = \sqrt{3}$

d)  $2^{x^3} = 256$ .

8. [20] Find  $dydx$ :

a)  $y = 2^{x \sin x}$

c)  $y = \arctan(\tan x)$

b)  $y = (1 + x)^{\ln x}$

d)  $y = \ln((1 + x)(1 + x^2))$ .

9. [20] Evaluate the following integrals:

a)  $\int x\sqrt{1 + x^2} dx$

c)  $\int x^4 \sin(x^5) dx$

b)  $\int (e^{\tan x} \sec^2 x) dx$

d)  $\int \frac{x}{1 + x^2} dx$ .

10. [20] a) Calculate the definite integral  $\int_{-\pi/2}^{\pi/2} \sin x dx$ .

b) Find the area under the curve  $y = \sin x$  from  $x = 0$  to  $x = \pi/2$ .